

The Role of Discourse in Inclusive and Equitable Mathematics Learning and Teaching

Cheryl Tobey, M.Ed.

Introduction

Discourse is a fundamental aspect of learning and understanding mathematics at all grade levels and for all students. According to the National Council of Teachers of Mathematics (NCTM), "Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication" (NCTM, 2014, p. 29).

This paper explores the what, why, and how of math discourse, emphasizing its crucial role in fostering deep understanding, enhancing argumentative skills, and promoting a culture of curiosity and active learning.

ABOUT THE AUTHOR



Cheryl Tobey, M.Ed., specializes in differentiated professional development for teaching students who struggle with mathematics. The author of Math Probes formative assessment activities in *Classroom Assessment Techniques*, she is an expert in identifying student misconceptions and developing learning targets to help define formative assessment.

- Professional Development Specialist, Education Development Center and Maine Mathematics and Science Alliance
- State Elementary Mathematics Specialist, Maine Department of Education
- Coauthor of 12 books on formative assessment

Table of Contents

The Introduction of Mathematics Curriculum.....	3
Developing New Curriculum Ideas-Field Development.....	5
Reading and Strategy Facilitate Mathematics Curriculum.....	6
Curriculum and Learning in Language Development.....	9
Curriculum and Formative Assessment.....	11

T I a Ma a a D

When the arc a e d c r e, he arc a e her h gh r c e e, cha e ge her
a d b d each her dea. Th erac e a r ach e e e her
der a d g f a he a ca c e b a d e e c r ca h g a d c ca

A a a a . I ead f a e
rece g f r a f r he eacher, de ac e e gage he ear g r c e b har g
her dea, e g a a d e r g d ere er ec e. Th de -ce ered
a r ach r e de e er der a d g a d f er a e e f er h a d age c ear g.
When de ee her dea a ed a d d c ed, he are re e a er a e e,
a d ee e ear g r e e.

T , a a a a a a a a
a a a a a . Fre a e, he ra(ragTh re)3.2(d cr be)TJ T[(

C 2023 California Mathematics Framework : SMP

The Standard for Mathematical Practice (SMP) embedded throughout the framework has been designed to help students develop mathematical practices that will help them become effective problem solvers, communicators, and critical thinkers. Each standard is designed to be a practice that is embedded throughout the SMP.

Mathematical Practice A: Problem Solving, Reasoning, and Communication, 1-23

The SMP standards are critical to the development of mathematical proficiency. They are found throughout the framework and are designed to be integrated with the content standards. (National Governors' Association Center for Best Practices and Council of Chief State School Officers, 2010). Mathematical proficiency has been defined as the ability to use the SMP. It is encouraged that students be able to apply the SMP in a variety of contexts, including those that are not explicitly stated in the framework.

Developing a deep understanding of the change of direction in a relationship between variables and the ability to represent a relationship between variables, a graph, and a table are all important skills for students to develop. The ability to represent a relationship between variables in a table, graph, and equation is a key skill for students to develop. The ability to represent a relationship between variables in a table, graph, and equation is a key skill for students to develop. The ability to represent a relationship between variables in a table, graph, and equation is a key skill for students to develop.

Talk

Display the *Talk About It!* *Designing Games* slide. Use the following questions to engage and check understanding.

ETP Pose Purposeful Questions

- How is comparing the ratios based on the first term different from comparing the ratios based on the second term?
- What other tools could be used to compare ratios?

Access Content

If students are unsure of how to compare ratios, they can explain that they can determine equivalent ratios that have the same value for the first term or can determine equivalent ratios that have the same value for the second term.

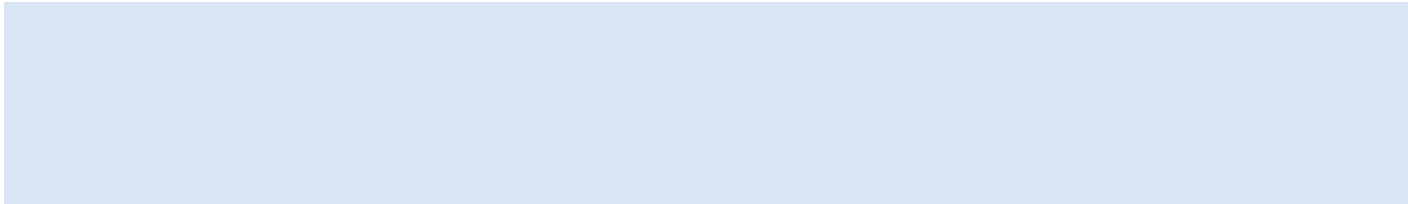
Math is... Choosing

- Why is this a good tool to represent the problem? Why?
- How does the table help you organize the information?

Listen to students' reasoning about how they:

- use tables to show each game.
- use multiplication to find equivalent ratios.
- multiply the terms in each ratio by a number such that the first terms of each ratio are the same.
- multiply the terms in each ratio by a number such that the second terms of each ratio are the same.

Example from *California
eMATH[®] Grade 6
Teacher Ed*



C 2023 Ca a Ma Fa :Ca N

E ab h g ca r r a dr e ca r de a e d g a d a g
. e . e f her eer. ' a he a ca dea a d e a . ha e a her?

Pr. d. g. r. e. f. r. de. a. ear. a. d. fe. ab. her. a. h. de. a. cr. ca. f. r. de. e. g. der. a. d. g. a. d. er. h. f. her. ear. g. B. e. c. rag. g. de. . hare. her. h. gh. fr. he. beg. . g. f. he. e. , each. ca. crea. e. a. e. r. e. here. e. er. ce. a. ed. a. d. a. he. a. ca. rea. . g. c. . e. red. Ear. d. c. . ca. be. e. ed. a. f. gh. dr. af. h. . g. a. . de. . hare. a. d. d. c. . her. a. , . hed. de. a. h. ch. are. a. er. re. ed. hr. gh. eer. feed. bac. a. d. c. ab. ra. e. re. (Ja. e. , 2023). Fre. e. r. e. f. r. d. c. r. e. hr. gh. he. e. he. de. . re. e. her. de. a. , ca. r. f. . der. a. d. g. , a. d. de. e. her. c. ce. a. der. a. d. g.

T. c. a. e. a. e. gag. ga. de. ec. e. a. he. a. ca. d. c. r. e. e. r. e. , ed. ca. r. ca. e. e. e. rac. ce. f. r. . g. de. re. . e. h. r. c. red. ech. e. e. a. h. a. e. a. d. . r. e. eer. d. c. . r. e. .

P a - a

S. ha. d. Se. (2011) . ed. e. e. rac. ce. f. r. e. ec. e. . g. de. re. . e. h. e. ca. d. c. . :

1. A. c. a. e. e. a. . de. re. . e. bef. re. he. e. . beg. .
2. Ac. e. r. . de. r. a. de. gage. e. d. r. g. he. e. .
3. Se. ec. ec. c. de. . re. e. her. a. he. a. ca. r. .
4. S. ra. eg. ca. e. e. ce. he. re. . e. f. r. he. d. c. .
5. Ma. e. c. ec. . be. ee. d. ere. . de. . re. . e. a. d. . he. ere. . e. e. a. he. a. ca. c. ce. .

The. e. rac. ce. . r. each. . f. er. gr. ch. a. he. a. ca. d. c. r. e. b. e. c. rag. g. de. e. r. de. e. gage. e. a. d. der. a. d. g. B. a. c. a. gre. . e. , each. ca. be. er. re. are. f. r. d. er. e. er. ec. e. , h. e. . ga. . he. . ga. ge. c. re. he. . rea. e. Se. ec. g. a. d. e. e. c. gre. . e. . a. e. . de. c. r. b. . b. a. . crea. e. a. c. he. e. . ar. a. e. d. c. . , . a. e. he. . g. de. . a. e. c. ec. . e. a. he. a. ca. de. a. Th. r. c. red. a. r. a. che. . er. ed. ca. r. . fac. a. e. r. d. c. e. c. er. a. . ha. e. ha. ce. ear. . g. . c. e. a. d. r. . e. a. c. ab. ra. e. ca. r. . c. re.

Fa a

The. e. f. r. c. red. ech. e. . cha. a. h. a. e. (Cha. e. a. , 2022). . r. he. r. ce. . f. har. ga. d. re. . g. de. a. Ma. h. a. e. refer. . ec. . ra. e. ge. r. ech. e. . fac. a. e. . ea. . g. f. a. he. a. ca. d. c. r. e. . ed. b. b. h. he. each. a. d. de. . Here. are. . e. c. . a. h. a. e. :

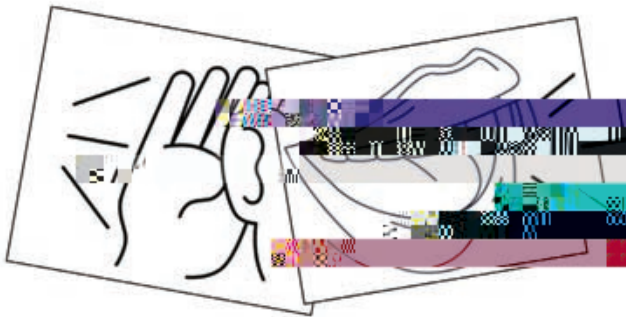
R ca. beg. . h, [S. , . re. a. . g. ha. ...?" r. "Le. e. ee. fl. . der. a. d. Y. h. ha. ...?" Th. . e. c. r. . he. de. . de. a. a. d. ca. r. . her. . e. gage. h. . re. de. e. .

R a f e beg h, "Ca a ha aga.?" r "Wha d d[a her. de.] a?" Th. e he . e . re ha a . de . ha e heard a d . der. d he e dea be g d. c . ed.

P f e beg h, [Ca e a. h...?] r "Wha a e h. ha...?" Th. a r, ach he . de . ar c a e her h. gh. r, ce. e a de a . e her dea . re cr ca , f. er, g dee ere e gage e . h he a he a ca c. ce . be g d. c . ed.

P f e beg h hra e . e, "Wha d . ce ab. ...?" r "H. c. d . g re. ...?" Th. ech. e he . de . f. c. her h. ga de . re a he a ca dea . re e ec. e .

C f e beg h r. . e, "H. h. ar...?" r "Ca h. fa her a . re re e. ...?" Th. e e . he . de . a ec. ec . be ee d ere . a he a ca dea , re re e. a . , r. ra ege .



Ta & L. e Card
(Kee e & T, be , 2017)

P a
Peer d. c . . r. de . de . ha a f. r . hare her dea , cha e ge each . her. h. g, a d b d. her c. ec. e . edge. E e he . ge ear er. ca e gage . eer d. c . . , be e g fr. he . r. . ar c a e her h. gh. a d heard er. e er. ec. e . Th. e e f. r c red r. e . . r. eer . erac . . F. re a . e, T. r. a d Ta . . g Ta & L. e Card (Kee e & T, be , 2017) ca . . r K. dergar e c. er. a . , a h. ca h. d. g a d. a . g he card r. de . a a g bere . der. f her. e a de . ec a . d r. g he d. c . . Th. er. e e . re ha each. de ha a cha ce . ea a d . e , f. er, g e . abe ar c a . a de ha c. g he a . f her. a he a ca d. c. r. e.

C 2023 Ca a Ma F a : P R

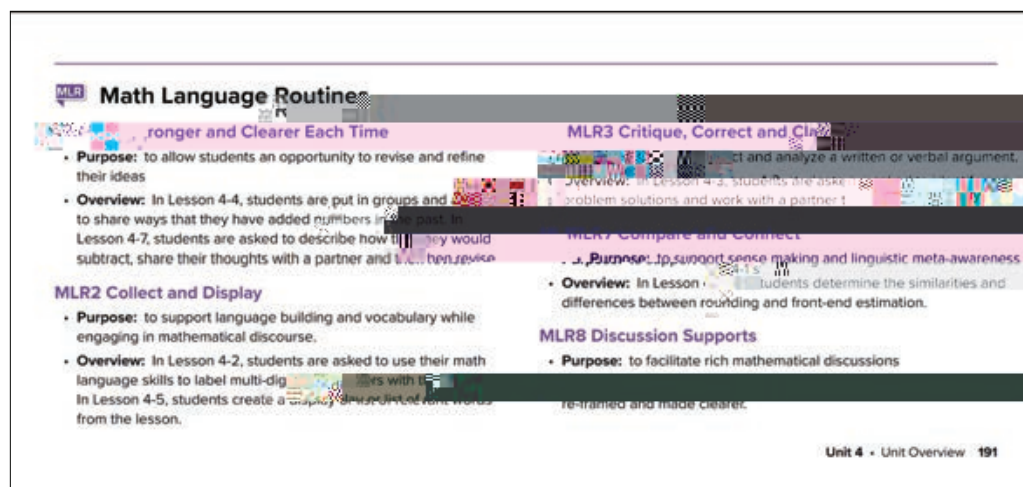
Peer re . c. g. a . erf r. e f. r r. . g. hared . der. a d g. f a he a c a . e 627.1201 T

Language Objectives

Language objectives become the focus of the lesson. They are designed to help students understand the content. For example, "By the end of the lesson, students will be able to explain the area of a rectangle." Language objectives are designed to be clear, measurable, and observable. They should be written in a way that is specific and measurable. For example, "Students will be able to explain the area of a rectangle." Language objectives should be written in a way that is specific and measurable. For example, "Students will be able to explain the area of a rectangle."

Strategies

Mathematical language routines (MLR) are designed to help students understand the content. They are designed to be clear, measurable, and observable. They should be written in a way that is specific and measurable. For example, "Students will be able to explain the area of a rectangle." Language objectives should be written in a way that is specific and measurable. For example, "Students will be able to explain the area of a rectangle."



Excerpt from *California evel Math® Grade 4 Teacher Edition*

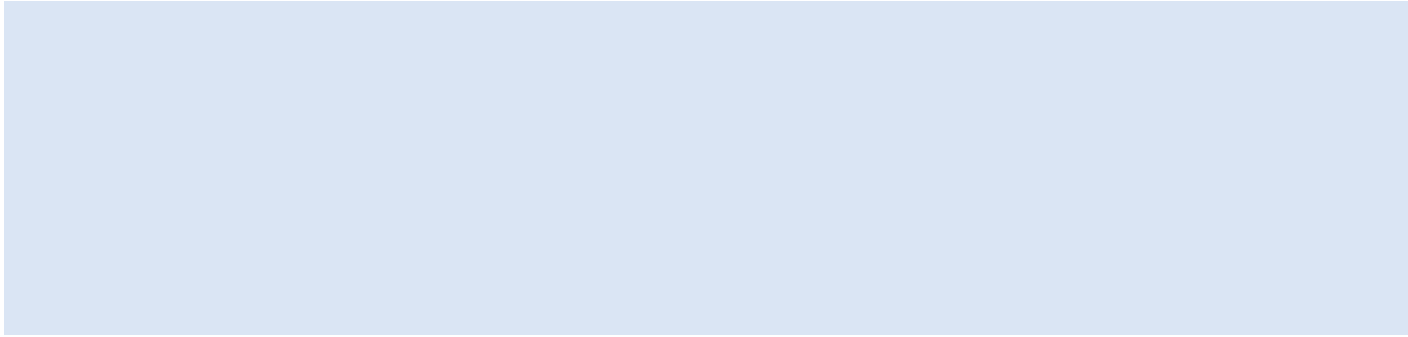
California Math Framework: Language Objectives

The framework provides language objectives for each grade level. For example, for Grade 4, the language objectives are:

Explain, Describe, and Read What a Problem Means (Mathematics, 19, 20)

The Math Language Routines, developed by Under the Big Tent Language of the Standards Center for Assessment, Learning, and Equity, provide each student with a framework for understanding the content. For example, "Students will be able to explain the area of a rectangle." Language objectives should be written in a way that is specific and measurable. For example, "Students will be able to explain the area of a rectangle."

Explain, Describe, and Read What a Problem Means (Mathematics, 49)



D a F a A

Bring It On!

Engage all students in a discussion about the key mathematical ideas in the lesson. Challenge students to think about the relationship between the two ratios.

ETP Elicit and Use of Evidence of Student Understanding

- How can you determine whether one ratio is equivalent to another ratio?
- How can you use a proportion to show that two ratios are equivalent?
- How is writing an equivalent ratio in different forms (fractions or decimals) helpful?

Listen to students' reasoning about how to

- use the process of creating equivalent ratios
- explain how to remove equivalent ratios.

Example from *California
Reveal Math® Grade 7
Teacher Edition*

C 2023 California Math Framework: Fractions

One of the things that a teacher should do is to have a good understanding of the different ways that students think about fractions. In order to do this, a teacher should have a good understanding of the different ways that students think about fractions. This is because students often have different ways of thinking about fractions, and it is important for a teacher to understand these different ways of thinking in order to be able to help students understand fractions better.

Mathematical Practices: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

Mathematical Practices: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18

R

Ball, H. W., Paac, N. A., Merrill, E. G., & Riddle-Kaufman, S. E. (2016). 5 Strategies for Scaling Mathematics for ELL. *Teaching Children Mathematics*, 23(2), 100-108.

California Department of Education. (2023). *California Health Care for* (2023 ed.). <https://www.cde.ca.gov/c/ah/fra/e/>

Cahana, Rebecca M., Melissa H. Hreha, and Jennifer B. Hreha. 2020. "Math Learning, Design Math: High School English Learners, Student-Led Discussion, and Math Tracking Literacy." *Mathematics Research Journal* 15(1): 82-103.

Carter, Michele and Jennifer M. Lager-O'Leary. 2018. "Using Culturally Diverse and a Transformative Framework to Address the Needs of English Learners." In *Edvard A. Sierra and Valerie L. Miller, Eds., A Fresh Look at Frameworks for Addressing the Needs of English Learners*, VA: National Council of Teachers of Mathematics.

Chang, S. H., O'Connor, C., & Anderson, N. C. (2022). *Mathematics: A Teacher's Guide for Using Culturally Diverse Mathematics, Grade K-6* (3rd ed.). Heinemann.

Hogarty, S. (2017). *Using Diverse Academic Language and Mathematics for English Learners*. Retrieved July 27, 2024, from <https://hoge.edweb.org/webinars/using-diverse-academic-language/>

Jain, A. (2020). *High Draft Math: Reaching Learners Who Struggle*. Pearson.

Kee, P., & Tabe, C. (2017). *Mathematics for All: 50 Strategies for Engaging All Learners*. Corwin.

National Council of Teachers of Mathematics. 2014. *Practices for Addressing Mathematics Success for All Learners*, VA: National Council of Teachers of Mathematics.

National Governors' Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). 2010. *Common Core State Standards*.

Shih, M. S., & Selig, M. K. (2011). *Effective Practices for Reaching and Educating All Learners*. National Council of Teachers of Mathematics.

Zerger, J., Decina, J., Rutherford-Quach, S., Darby, V., Sarrazin, R., Weis, S., & Maa, J. (2017, February 28). *Practices for the Design of Mathematics Curriculum: Practicing Language and Culture Development* (Version 2.0). Graduate School of Education.

Teachers are always learning:

a a / a

