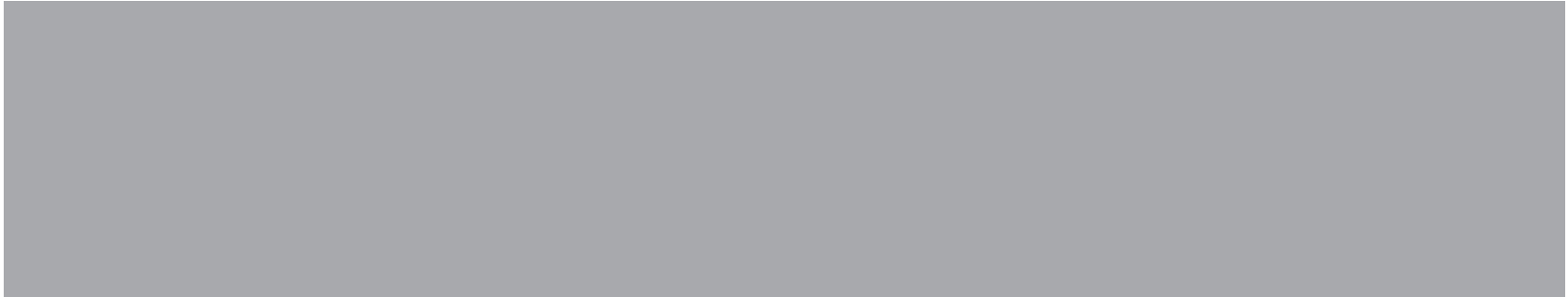Today, the most advanced educational technology solutions are digital programs that provide individualized, responsive learning experiences. These programs—known as adaptive learning systems—tailor activities to the educational needs of each student. Unlike previous software programs that merely provided positive or corrective feedback to each student's answers, adaptive learning systems offer real-time, individualized instruction as well as feedback. Specifically, adaptive learning systems deliver content that depends directly on the students' current levels of knowledge and skill. Because these systems use algorithms to interpret students' responses, they continually adapt to students' skill development. They "identify skills that . . . students have mastered, diagnose instructional needs, monitor academic growth over time, make data-driven decisions at the classroom, school, and district levels, and place students into appropriate instructional programs" (Nedungadi and Raman 2012, 662). For these reasons, adaptive learning systems are quickly becoming integral parts of regular education and intervention programs.

Learning Trajectories

Learning trajectories are theoretical models that describe in detail how students learn particular concepts. Based on extensive observational and experimental research, these learning trajectories illuminate the critical developmental sequences that children undergo as they build conceptual knowledge and skills. For example, the following model (Sarama and Clements, JECR, 2002) highlights seven distinct developmental stages in the learning of shape composition:

1. Pre-Composer. Manipulates shapes as individuals, but is unable to combine them to compose a larger shape.

2. Piece Assembler. Similar to level 1, but can concatenate shapes to form pictures. In free-form "make a picture" tasks, for example, each shape used represents a unique role, or function in the picture. Can fill simple frames using trial and error. Uses turns or flips to do so, but again by trial and error; cannot use motions to see shapes from different perspectives. Thus, children at levels 1 and 2 view shapes only as wholes and see no geometric relationship between shapes or between parts of shapes (i.e., a property of the shape).

3. Picture Maker. Can concatenate shapes to form pictures in which several shapes play a single role, but uses trial and error and does not anticipate creation of a new geometric shape. Chooses shapes using gestalt configuration or one component such as side length. If several sides of the existing arrangement form a partial boundary of a shape (instantiating a schema for it), the child can find and place that shape. If such cues are not present, the child matches by a side length. The child may attempt to match corners, but does not possess angle as a quantitative entity, so will try to match shapes into corners of existing arrangements in which their angles do not fit. Rotating and flipping are used, usually by trial-and-error, to try different arrangements (a "picking and discarding" strategy). Thus, can complete a frame that suggests placement of the individual shapes but in which several shapes together may play a single semantic role in the picture.

4. Shape Composer. Combines shapes to make new shapes or fill frames, with growing intentionality and anticipation ("I know what will fit"). Chooses shapes using angles as well as side lengths. Eventually considers several alternative shapes with angles equal to the existing arrangement. Rotation and flipping are used intentionally (and mentally, i.e., with anticipation) to select and place shapes. Can fill complex frames or cover regions. Imagery and systematicity grow within this and the next levels. In summary, there is intentionality and anticipation, based on shapes' attributes, and thus, the child has imagery of the component shapes, although imagery of the composite shape develops within this level (and throughout the next levels).

5. Substitution Composer. Deliberately forms composite units of shapes[,] and recognizes and uses substitution relationships among these shapes (e.g., two pattern block trapezoids can make a hexagon).

6. Shape Composite Iterater. Constructs and operates on composite units intentionally. Can continue a pattern of shapes that leads to a "good covering," but without coordinating units of units.

7. Shape Composer with Units of Units. Builds and applies units of units (superordinate units). For example, in constructing spatial patterns, children extend their patterning activity to create a tiling with a new unit shape—a (higher-order) unit of unit shapes that they recognize and consciously construct; that is, children conceptualize each unit as being constituted of multiple singletons and as being one higher-order unit.

Of course, developmental models describe widely seen stages and patterns of development, but researchers and educators acknowledge significant variations across individuals. Therefore, effective learning programs make use of learning trajectories, while simultaneously responding to students' particular levels of development. As Douglas Clements states, the term developmentally appropriate "means challenging but attainable for most children of a given age range, flexible enough to respond to inevitable individual variation, and, most important, consistent with children's ways of thinking and learning" (2002, 161).

Computer Assisted Intervention (CAI) and Manipulatives

Two powerful tools that are helping children learn mathematics are Computer-Assisted Intervention (CAI)—based on adaptive learning systems—and manipulatives. Already, research has shown that students make significant gains using a CAI program for as little as ten minutes a day (Fletcher, Hawley, & Piele 1990). For example, one study showed that children's attitudes towards mathematics became more positive because they used CAI (Huang, Liu, & Chang 2012). According to the authors, the children were motivated because the program provided them with a personalized context. This same study also showed that the children using the CAI program had significantly higher scores.

Another study showed that preschool- and primary-grade children made the greatest gains when using a CAI program. This improvement was particularly noticeable with children in compensatory education programs (Sarama and Clements 2002). However, when CAI served as the primary mode of instruction, students did not learn the conceptual ideas behind the mathematics. In other words, the study suggested that CAI is most useful when blended with another type of learning (Sarama and Clements 2006).

Manipulatives have long been recognized as effective tools for teaching and learning. For example, educators often use manipulatives as a bridge to help students apply mathematical concepts to the real world (Durmu and Karakirik 2006). As Clements and McMillen state, "students who use manipulatives in their mathematics classes usually outperform those who do not" (1996, 270). To achieve the best outcomes, however, teachers must guide students' use of manipulatives, so students gain deeper insights into the concepts the manipulative-based activities represent. Without teacher guidance, children tend to develop only basic levels of conceptual understanding.

In recent years, manipulatives have become part of instructors' digital toolboxes. In fact, students can now work with digital manipulatives just as they work with traditional manipulatives—counting them, stacking them, rotating them, comparing them, sorting them, and building with them. However, digital manipulatives have additional benefits: students can visualize and move them in ways that are not possible with physical traditional manipulatives. For example, while students must physically trade typical base-ten blocks to show regrouping—an obviously important skill needed for addition and subtraction—students can easily break apart digital blocks to demonstrate regrouping (Sarama and Clements 2006). "Such actions are more in line with the mental actions that we want students to carry out. The computer also connects the blocks to the symbols" (Sarama and Clements 2006, 113).

Building Blocks: Combining Learning Trajectories with Adaptive Learning

Building Blocks—an integral, digital component of Number Worlds—relies directly upon well-documented learning trajectories of mathematical concepts as well as the ability to respond flexibly to students' inputs and current levels of understanding. Building Blocks is an adaptive learning program that blends CAI with the use of digital manipulatives—a combination shown to promote greater success than either CAI or manipulatives used independently. In addition, since teacher-led instruction helps students maximize their learning, Building Blocks is an important component of the Number Worlds curriculum. Through engaging scenarios and games, Building Blocks "connect[s] children's informal knowledge to more formal school mathematics. The result is a package that . . . motivat[es] . . . children, but is also comprehensive. . . . [It uses] exploratory environments that include specific tasks and guidance, building concepts[,] and well-managed practice [in] building [mathematical] skills, a full set of critical curriculum components, and a full range of mathematical activities" (Sarama 2004, 373).

Research on the effectiveness of Building Blocks in the classroom has clearly shown Building Blocks to be a very successful adaptive learning system. In multiple studies, "Building Blocks classrooms significantly outperformed control classrooms on tests of number and geometry (including measurement, patterning, and so on), with effect sizes from 1 to 2 standard deviations, up to double what is considered a strong effect" (Sarama and Clements 2006, 126). These strong, positive effects show achievement gains near or approximately equal to those recorded for individual tutoring (Clements and Sarama 2007). Therefore, teachers trying to provide effective and efficient intervention in math will undoubtedly find Building Blocks and the entire Number Worlds program to be important assets in their mathematics curricula.

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