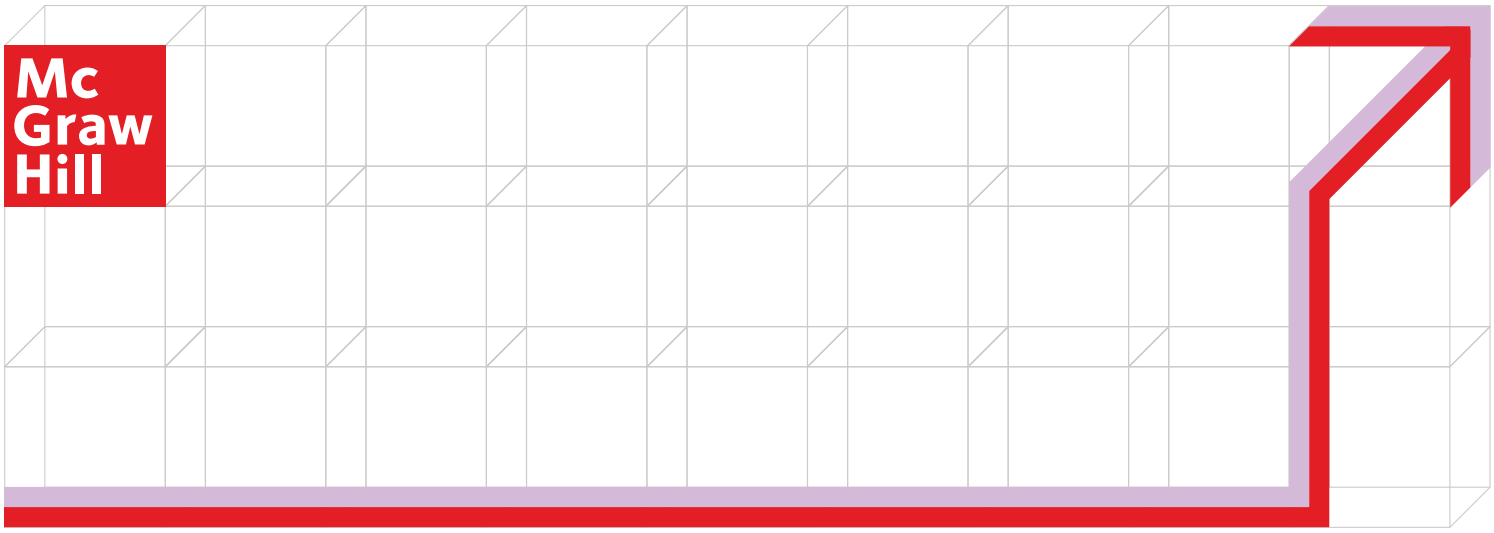

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S a a l a Ma a a P a

P a 1: Ma a (MP 1)

Similarly, students in sixth grade are able to decide that 750 ÷ 250 doesn't necessarily need an algorithm if they understand benchmark numbers or recognize the relationship to 75 ÷ 25. Moreover, this practice requires students to understand tools and how they work. This means that students understand that rulers measure the distance between two points and measuring with them does not have to begin with zero.

P a 6: A , , P , , (MP 6)

Precision with computation and communication is at the core of MP 6. Precision with calculation increases as students develop and refine strategies for e□cient computation as well as a sense of reasonableness. Communicating precisely means that students use mathematics terms correctly. It also means that they communicate precise solutions noting correct units of measure. As with the other practices, greater precision comes with experience and opportunity on a frequent and consistent basis. Therefore, students need daily experiences to acquire and use mathematics vocabulary and work with operations and number relationships.

P a 7: L , , F , a , Ma S , , S (MP 7)

Patterns and structure in mathematics help us understand quantity and concept. MP 7 calls for students to look for and make sense of structures in mathematics. A foundational example of structure is inherent in base ten place value. Another example is within the properties of operations. For example, the distributive property of multiplication is a structure that enables students to leverage partial products when multiplying multidigit numbers. Exploration and discussion are critical for exposing and understanding the structure of mathematics. Recording patterns so that students clearly see relationships and make generalizations also plays an important role in this practice. This practice also calls for students to see complicated things as a collection of things. "For example, they can see 5 − 3(x − y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y" (CCSCO, 8).

P a 8: L , , F , a , Ma U , R a R a , , , (MP 8)

Finding and using patterns is an e□ective problem-solving strategy. MP 8 comes to life as students see and leverage relationships between problems and calculations. For example, students who exhibit proficiency with MP 8 conclude that 5 × 8 = 40 because they know that 5 × 7 = 35 and that 5 × 8 is one more group of 5 (Illustrative Mathematics, 2014). In other situations, students might transfer understanding of combinations of 10 (e.g., 4 + 6, 5 + 5, 7 + 3) to quickly find the sum of 40 and 60 or 400 and 600. In eighth grade, students might use repeated reasoning with right triangles to discover and use ideas about 3:4:5 triangles. Repeated reasoning enables students to take shortcuts grounded in understanding. Implications for instruction are similar to other practices in that intentional selection of problems and computations, exploration, and discussion must be in place.

W D, Ma a a P a Ma ?

The math practices and their predecessors have always mattered. Advances in technology influence the changing needs of employers and society in general (Boaler, 2015). The need for critical-thinking, problem-solving, and communication skills, as well as abilities to use tools e□ectively, seek evidence, and make sense of patterns, has never been greater. Evidence of this need is reflected in other disciplines. The Next Generation Science Standards Lead States (2013) outline practices that embody similar behaviors. Language arts also calls for practices or habits of mind outlined in "student capacities" (CCSSO, 2010). Today, learning to do mathematics is more than a set of procedures that one learns and uses without deep thinking. Instead, students learn mathematics content and mathematics practices so that they are armed with skills to prepare them for their future.

M a, Fa A, Ma a a P a

There is a cloud of myths about teaching and learning mathematics. There are notions that men are better at mathematics than women. There is the notion that there is a "best" way to do mathematics or that it's bad to count on one's fingers (University of Fairbanks, 2018). These beliefs are false. Naturally, there are myths about the MPs (Mateas, 2016). Some of those myths include:

1. **T Ma a a P a ,,' a a .**
   We must prepare students for their future, yet there are many jobs that exist today that won't by the time our elementary students enter the workforce (Forbes, 2018). It is di□cult to imagine that people once thought that elevator operators and telephone operators were irreplaceable. Changes are greatly influenced by technology, and in today's world of ever-advancing technology, thinking may be the most important skill we can develop in our students. MPs are mathematical habits of mind. They are the behaviors of mathematicians and thinkers. They may matter even more than the mathematics content that can be learned by watching a video or reading a blog.
2. **Y, , a , ,,.**

in many lessons. Some MPs require a more intentional evocation. What is most important is that teachers call attention to these practices and evidence them as students engage in mathematics so that the practices are better understood and ultimately used.

4. **P a a a .**
These practices are complex. As with any complex skill or concept, they cannot be taught and mastered in a single class period. Instead, they are nurtured over long periods of time. Instances of those practices should be highlighted and reflected upon. Revisiting practices throughout a week or unit helps students better understand them. It helps students recognize that careful selection of tools extends beyond a study of measurement and that reasoning about symbols and quantities is a daily experience.

5. **O a , a a a .**
The math practices should come about through rich problems and quality tasks. Clearly, a page of 40 problems requiring nothing more than rote procedure is unlikely to elicit these practices. However, an eccentric prompt and complicated tasks aren't required either. There is no one special task that causes students to master constructing viable arguments. Rather, it happens over time through a collection of authentic tasks that consistently ask students to explain how they represented problems, how they solved problems, why their solutions are accurate, or how they made connections between numbers, representations, and operations.

6. **P a a a a .**
The MPs are unique yet interconnected to one another. They are also closely connected to the mathematics content that is taught. Content such as place value, two-dimensional figures, or fractions is the what of mathematics instruction. at of ted t5yt symbdysat oapace vesi(athome)1 (, aents to r

(PARCC, 2018; Smarter Balanced, 2018). Other tests do not report performance with practices. Even so, the skills and behaviors of these practices are incorporated into assessment items and design.

9. **T a a , ' , a , a .**
"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSO, 2010). The practices are curriculum expectations in all grades. All elementary students are to gain experience with and develop behaviors inherent in these practices. Certain practices such as solving problems, constructing arguments, and attending to precision are obviously present in these grades; the others must be as well. Students can make sense of symbols and equations when connected to their representations of problems. Students can make decisions about how tools help them and when tools are unnecessary. Students can see patterns within numbers and results to find e□ciencies. Simply, the content may be di□erent from grade to grade, but these practices remain a constant.

H , D , W B , , P a , L ?

The practices won't magically appear in a mathematics lesson. They have to be understood and planned for intentionally. They must be discussed and reflected upon. They come to life when we:

- Understand what they mean.
- Recognize what they look like.
- Plan for them intentionally and daily, though they are not a list to be checked o□.
- Select authentic tasks that elicit behaviors that are evidence of the practices and spark curiosity.
- Discuss how the practices were evident in a lesson or investigation by reflecting and discussing them.

W a I E , , P a , a Ma a L , ?

There will be evidence that the practices are alive and well in a lesson or classroom, but is important to keep in mind that these classrooms can look di□erent in respect to how students are engaging in the di□erent processes. Teachers can find evidence in artifacts of a lesson plan or in the writing and representations of student work. Teacher and student behaviors are strong indicators of the practices as well because student attitudes, perspectives, and behaviors are related to what teachers do and say (Northern, 2017). In essence, student behaviors are not random.

The charts below can be useful for teachers and students as they describe and define the practices. They can help students reflect on how they behaved like a mathematician during the lesson. Administrators might use the descriptors as indicators on a classroom walk-through or to buoy feedback to teachers. The charts are not exhaustive.

Mathematical Practice 4: Model with mathematics.

Students	Teachers
▪ Use physical tools to make predictions and solve problems ▪ Use drawings and diagrams to make predictions and solve problems ▪ Use tables and graphs to predict and solve problems	▪ Make tools available for students to use ▪ Highlight how tools, drawings, diagrams, tables, and equations support reasoning and solutions ▪ Use intentional and appropriate representations

Mathematical Practice 5: Use appropriate tools strategically.

Students	Teachers
▪ Understand how tools work ▪ Determine when tools are helpful ▪ Can describe why they used a tool for a given situation ▪ Evaluate whether the results generated with a tool are reasonable	▪ Make tools available to students ▪ Use tools during instruction ▪ Focus on how tools work as well as how to use them ▪ Discuss whether tools are necessary for a given situation ▪ Discuss whether results are reasonable

Mathematical Practice 6: Attend to precision.

Students	Teachers
▪ Use vocabulary correctly ▪ Calculate precisely ▪ Specify units of measure ▪ Use appropriate symbols	▪ Use vocabulary correctly ▪ Model e□cient strategies for computation, noting accurate solutions ▪ Highlight units of measure and symbols ▪ Correct inaccuracies

Mathematical Practice 7: Look for and make sense of structure.

Students	Teachers
▪ Look for patterns in numbers and operations ▪ Use structure to compute and solve	

7. Record related equations in ways so that patterns are easily observed (MP 7).

8. Representations used must align to the mathematics. Equal group multiplication problems should be represented with equal groups rather than area models.

Q & A

- MP 1: Do my students engage in problems daily? Do my students have diverse strategies for solving problems?
- MP 2: Do my students have opportunities to reason about numbers?
- MP 3: Do my students justify their thinking? Do they communicate their ideas clearly? Do they passively agree with classmates?
- MP 4: Do my students construct models of problems and situations with physical tools, drawings, and equations?
- MP 5: Do my students know how to use tools?
- MP 6: Do my students use correct vocabulary, attend to unit labels, and recognize precision?
- MP 7: Do my students have opportunities to observe, discuss, and make sense of structures in mathematics?
- MP 8: Do my students explain why shortcuts work?

R

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