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Building Blocks

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This study evaluated the efficacy of a preschool mathematics program based on a comprehensive model of developing research-based software and print curricula. *Building Blocks,* funded by the National Science Foundation, is a curriculum development project focused on creating research-based, technology-enhanced mathematics materials for pre-K through grade 2. In this article, we describe the underlying principles, development, and initial summative evaluation of the first set of resulting materials as they were used in classrooms with children at risk for later school failure. Experimental and comparison classrooms included two principal types of public preschool programs serving low-income families: state funded and Head Start prekindergarten programs. The experimental treatment group score increased significantly more than the comparison group score; achievement gains of the experimental group approached the sought-after 2-sigma effect of individual tutoring. This study contributes to research showing that focused early mathematical interventions help young children develop a foundation of informal mathematics knowledge, especially for children at risk for later school failure.

Key words: Computers, Curriculum, Early childhood, Equity/diversity, Instructional intervention, Instructional technology, Preschool/primary, Program/project assessment

Curricula are rarely developed or evaluated scientifically (Clements, 2007). Less than 2% of research studies in mathematics education have concerned the effects of textbooks (Senk & Thompson, 2003). This study is one of several coordinated efforts to assess the efficacy of a scientifically based curriculum; specifically, whether a preschool mathematics curriculum was developing the mathematical knowledge of disadvantaged 4-year-old children (Clements, 2002; Clements & Battista, 2000).

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Building Blocks is a NSF-funded pre-K to grade 2 mathematics curriculum development project designed to comprehensively address recent standards for early

ment of *mathematical* activity. To do so, the materials integrate three types of media: computers, manipulatives (and everyday objects), and print. Pedagogical foundations were similarly established; for example, we reviewed research using computer software with young children (Clements, Nastasi, & Swaminathan, 1993; Clements & Swaminathan, 1995; Steffe & Wiegel, 1994). This research showed that computers can be used effectively by children as young as 3 or 4 years of age and that software can be made more motivating and educationally effective by, for example, using animation and children's voices and giving simple, clear feedback.

The phase of *Subject Matter A Priori Foundation* was used to determine subject matter content by considering what mathematics is culturally valued (e.g., NCTM, 2000) and empirical research on what constituted the core ideas and skill areas of mathematics for young children (Baroody, 2004; Clements & Battista, 1992; Fuson, 1997), with an emphasis on topics that were mathematical foundational, generative for, and interesting to young children (Clements, Sarama, et al., 2004). One of the reasons underlying the name we gave to our project was our desire that the materials emphasize the development of basic *mathematical building blocks* (the second meaning of the project's name)—ways of knowing the world mathematically—organized into two areas: spatial and geometric competencies and concepts, and numeric and quantitative concepts. Research shows that young children are endowed with intuitive and informal capabilities in both these areas (Baroody, 2004; Bransford, Brown, & Cocking, 1999; Clements, 1999; Clements, Sarama, et al., 2004). Three mathematical themes are woven through both these main areas: patterns, data, and sorting and sequencing. For example, challenging number activities do not just develop children's number sense; they can also develop children's competencies in such logical competencies as sorting and ordering (Clements, 1984).

Perhaps the most critical phase for *Building Blocks* was *Structure According to Specific Learning Model.* All components of the *Building Blocks* project are based on learning trajectories for each core topic. First, empirically based models of children's thinking and learning are synthesized to create a developmental progression of levels of thinking in the goal domain (Clements & Sarama, 2004b; Clements, Sarama, et al., 2004; Cobb & McClain, 2002; Gravemeijer, 1999; Simon, 1995). Second, sets of activities are designed to engender those mental processes or actions hypothesized to move children through a developmental progression. We present two examples, one in each of the main domains of number and geometry.

The example for number involves addition. Many preschool curricula and practitioners consider addition an inappropriate topic before elementary school (Clements & Sarama, in press; Heuvel-Panhuizen, 1990). However, research shows that children as young as toddlers can develop simple ideas of addition and subtraction (Aubrey, 1997; Clements, 1984; Fuson, 1992a; Groen & Resnick, 1977; Siegler, 1996). As long as the situation makes sense to them (Hughes, 1986), young children can directly model different types of problems using concrete objects, fingers, and other strategies (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). Such early invention of strategies, usually involving concrete objects and based on subitizing and counting, plays a critical developmental role, as the sophisticated counting and composition strategies that develop later are all abbreviations or curtailments of these early solution strategies (Carpenter & Moser, 1984; Fuson, 1992a).

Most important for our purpose, reviews of research provide a consistent developmental sequence of the types of problems and solutions in which children can construct solutions (Carpenter & Moser, 1984; Clements & Sarama, in press; for the syntheses most directly related to our work, see Clements, Sarama, et al., 2004; Fuson, 1992a). Selected levels of the resulting addition learning trajectory are presented in Figure 1. The left column briefly describes each level and the research supporting it. The middle column provides a behavioral example illustrating that level of thinking. The learning trajectory continues beyond the last row in Figure 1 (details are in the *Building Blocks* curriculum and other sources, Clements & Sarama, 2007; Clements, Sarama, et al., 2004).

The next step of building the learning trajectory is to design materials and activities that embody actions on objects in a way that mirrors what research has identified as critical mental concepts and processes—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks are instantiated in on- and off-computer activity as actions (processes) on objects (concepts). For example, children might create, copy, and combine discrete objects, numbers, or shapes as representations of mathematical ideas. Offering students such objects and actions is consistent with the Vygotskian theory that mediation by tools and signs is critical in the development of human cognition (Steffe & Tzur, 1994). Further, designs based on objects and actions force the developer to focus on explicit actions or processes and what they will mean to the students. For example, on- and off-computer activity sets such as "Party Time" have the advantage of authenticity as well as serving as a way for children to mathematize these activities. In one of the "Party Time" activities involving setting the table, children use different mathematical actions such as establishing one-to-one correspondence, counting, and using numerals to represent and generate quantities to help get ready for a party. For these and other activities, the tasks themselves are often variations of those common in educational curriculum; what is unique in these cases is the more detailed consideration of actions on objects, the placement of the tasks in the research-guided learning trajectories, and the use of software.

For the addition trajectory, at the *Nonverbal Addition* level, children work on a software program in which they see three toppings on a pizza, then, after the top of the box closes, one more being placed on the pizza. Children put the same number of toppings on the other pizza (see the right column in the first row of Figure 1). The teacher conducts similar activities with children using colored paper pizzas and manipulatives for toppings. Similarly, the Dinosaur Shop scenario is

these and other tasks for the levels described in Figure 1 are single items, groups of items, and numerals. The actions include creating, duplicating, moving, combining, separating, counting, and labeling these objects and groups to solve tasks corresponding to the levels. The unique advantages of the software contexts include making these actions explicit, linking representations (computer manipulatives, spoken number words, and numerals), providing feedback, and guiding children along the research-based learning trajectories (e.g., moving a level forward or backward depending on a children's performance).

An example in geometry involves shape composition (other domains were shapes and their properties, transformations/congruence, and measurement, all determined through consensus building, see Clements, Sarama, et al., 2004). The composition of two-dimensional geometric figures was determined to be significant for students in two ways. First, it is a basic geometric competence, growing from preschoolers' building with shapes to sophisticated interpretation and analysis of geometric situations in high school mathematics and above. Second, the concepts and actions of creating and then iterating units and higher-order units in the context of constructing patterns, measuring, and computing are established bases for mathematical understanding and analysis (Clements et al., 1997; Reynolds & Wheatley, 1996; Steffe & Cobb, 1988). The domain is significant to research and theory in that there is a paucity of research on the trajectories students might follow in learning this content.

The developmental progression was born in observations of children's explorations (Sarama, Clements, & Vukelic, 1996) and refined through a series of clinical interviews and focused observations (leading to the learning trajectory summarized in Figure 2, adapted from Clements, Wilson, & Sarama, 2004). From a lack of competence in composing geometric shapes (*Pre-Composer),* children gain abilities to combine shapes—initially through trial and error (e.g., *Picture Maker)* and gradually by attributes—into pictures, and finally synthesize combinations of shapes into new shapes (composite shapes). For example, consider the *Picture Maker* level in Figure 2. Unlike earlier levels, children concatenate shapes to form a component of a picture. In the top picture in that row, a child made arms and legs from several contiguous rhombi. However, children do not conceptualize their creations (parallelograms) as geometric shapes. The puzzle task pictured at the bottom of the middle column for that row illustrates a child incorrectly choosing a square because the child is using only one component of the shape, in this case, side length. The child eventually finds this does not work and completes the puzzle but only by trial and error.

A main instructional task requires children to solve outline puzzles with shapes off and on the computer, a motivating activity (Sales, 1994; Sarama et al., 1996). The software activity "Piece Puzzler" is illustrated in the third column in Figure 2 (on pages 144–145). The *objects* are shapes and composite shapes and the *actions* include creating, duplicating, positioning (with geometric motions), combining, and decomposing both individual shapes (units) and composite shapes (units of units). The characteristics of the tasks require actions on these objects corresponding to each level in the learning trajectory. Note that tasks in these tables are intended to support the developing of the *subsequent* level of thinking. That is, the instructional task in the *Pre-Composer* row is assigned to a child operating at the *Pre-Composer* level and is intended to facilitate the child's development of competencies at the *Piece Assembler* level.

Ample opportunity for student-led, student designed, open-ended projects are included in each set of activities. Problem posing on the part of students appears to be an effective way for students to express their creativity and integrate their learning (Brown & Walter, 1990; Kilpatrick, 1987; van Oers, 1994), although few empirical studies have been conducted, especially on young children. The computer can offer support for such projects (Clements, 2000). For "Piece Puzzler," students design their own puzzles with the shapes; when they click on a "Play" button, their design is transformed into a shape puzzle that either they or their friends can solve. In the addition scenarios, children can make up their own problems with pizzas and toppings, or dinosaurs and boxes.

Our application of formative evaluation phases 5–8 is described in previous publications (Sarama, 2004; Sarama & Clements, 2002). In brief, we tested components of the curriculum and software using clinical interviews and observations of a small number of students to ascertain how children interpreted and understood the objects, actions, and screen design. Next, we tested whether children's actions on objects substantiated the actions of the researchers' model of children's mathematical activity, and we determined effective prompts to incorporate into each level of each activity. Although teachers were involved in all phases of the design, in phases 7–8 we focused on the process of curricular enactment (Ball & Cohen, 1996), using classroom-based teaching experiments and observing the entire class for information concerning the usability and effectiveness of the software and curriculum. Finally, a content analyses and critical review of the materials at each stage of development was conducted by s byacti 62 (-2004summar58 (Prie -12 (Priegn,) -49 (Prie53htiv 0 Tc ET BT

Level	Examples (above, free-form pictures; below, puzzles)	Instructional task
Pre-Composer. Children manipulate shapes as individuals but are unable to combine them to compose a larger shape. In free form—"make a picture"—tasks, shapes often picture in middle do not touch (upper column). In puzzle tasks, shapes do not match simple outlines (lower picture in middle column). The instructional task (illustrated similar tasks on the computer in the last column; are presented with manipulatives and paper outlines or wooden form puzzles) uses outlines in which children can simply match shapes without turn or flip motions. (This and subsequent levels emerged from the same body of research, Clements, Wilson, et al., 2004; Mansfield & Scott, 1990; Sales & Hildebrandt, 2002; Sarama et al., 1996.)		
Piece Assembler. Children can place shapes contiguously to form pictures. In free-form tasks, each shape used represents a unique role or function in the picture (e.g., one shape for one leg).		

Level	Examples (above, free-form pictures; below, puzzles)	Instructional task
Shape Composer. Children combine shapes to make new shapes or		

Head Start (site 2) prekindergarten programs. State funded programs are urban programs in which most children receive free (63%) or reduced lunch (11%) and are 58% African American, 11% Hispanic, 28% White non-Hispanic, and 3% other. Head Start programs are urban programs in which virtually all children are qualified to receive free (97%) or reduced lunch (2%) and are 47% African American, 13% Hispanic, 30% White non-Hispanic, and 10% other. At each site, one classroom was assigned as experimental, one comparison. Both site 1 teachers had worked with us on the early development of the materials and were considered excellent teachers by their principal and peers. They agreed to have one selected

Instrument

The *Building Blocks Assessment of Early Mathematics, PreK–K* (Sarama & Clements, in press) uses an individual interview format, with explicit protocol,

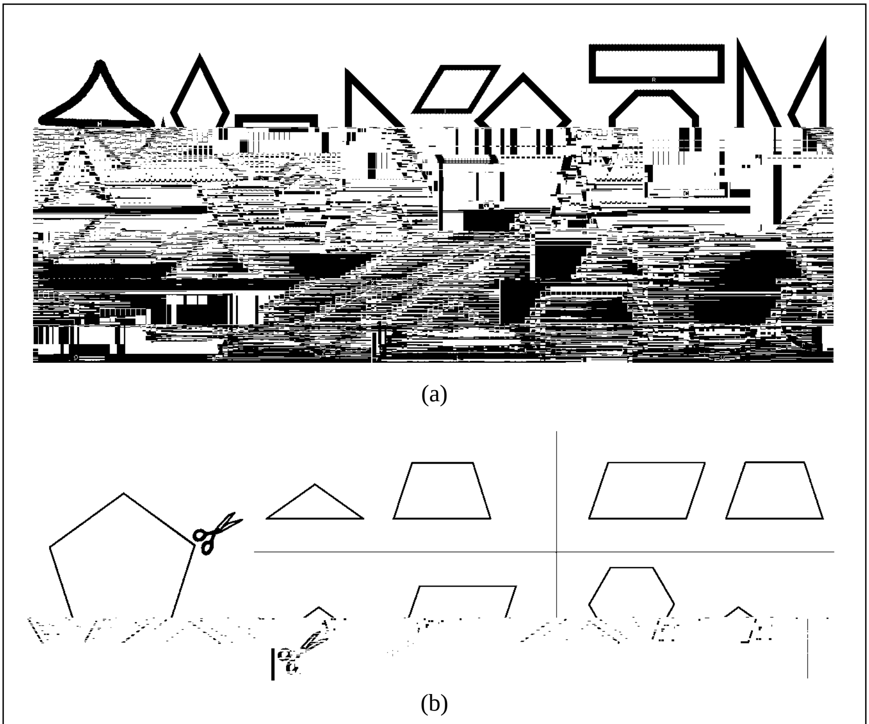

Figure 3. Sample geometry items from the Building Blocks Assessment of Early Mathematics, PreK–K (Sarama & Clements, in press). (a) Given the illustrated cut-out shapes, the child is asked to “Put only the triangles on this paper.” (b) The child is asked, “Pretend you cut this pentagon from one corner to the other. Which shows the two cut pieces?”

“Pizza Pizzazz” scenario includes activities on recognizing and comparing number, counting, and arithmetic. In the first three activities, children match pizzas with the same number of toppings (early number recognition and comparing), create a pizza with the same number of toppings as a given pizza (counting to produce a set), and create a pizza that has a given number of toppings given only a numeral (counting to produce a set that matches a numeral). Later activities in that setting involve addition (see the third column for *Nonverbal Addition* and *Find Change* in Figure 1). The software’s management system presents tasks, contingent on success, along research-based learning trajectories. Activities within various scenarios are introduced according to the trajectory’s sequences. Figure 1 illustrates, for example, how two activities from the “Dinosaur Shop” scenario are sequenced between two illustrated activities from the “Pizza Pizzazz” scenario. Off-computer activities, such as learning center activities, involve corresponding activities. For example, corresponding to the first “Pizza Pizzazz” activity, the teacher sets out a learning center by hiding paper “pizzas” with different numbers of toppings under several opaque containers and placing one such pizza with three toppings in plain view. Children lift each container and count the toppings until they find the matching pizza. They then show the teacher or other adult.

All participating teachers maintained their typical schedule, including circle (whole-group) time, work at centers, snack, outdoor play, and so forth for the 25 school weeks between pretesting and posttesting. The experimental teachers merely inserted the *Building Blocks* activities at the appropriate point of the day. For example, circle time might include a finger play that involved counting and a brief introduction to a new center or game. Center time would include individual work at the curriculum's software or learning centers, guided by the teacher or aide as they circulated throughout the room (specific suggestions for guidance are specified in the curriculum). As a specific example, children might be introduced to new puzzles such as those at the level of the *Picture Maker* level of Figure 2, then engage in physical puzzles with pattern blocks and tangrams in a learning center, or similar puzzles in the "Piece Puzzler" software activity. Teachers guided children by ison teacher used Creative Curriculum (Teaching Strategies Inc., 2001) as well as some "home-grown" curricular activities for mathematics. Visits to those classrooms indicated that each was following the curricula as written.

Analyses

To assess the effectiveness of the curriculum, we conducted factorial repeated measures analyses, with time as the within-group factor, and two between-group factors, school and treatment, evaluating differences in achievement from pre- to posttest on both tests (children did work in the same class, but the software and center activities were engaged in individually, so the child was used as the unit of analysis). In addition, two effect sizes were computed for each test. We compared experimental posttest (E2) to the comparison posttest (C2) scores as an estimate of differential treatment effect. We also compared experimental posttest to experimental pretest (E2 to E1) scores as an estimate of the achievement gain within the experimental curriculum. Effect sizes were computed using adjusted pooled standard deviations (Rosnow & Rosenthal, 1996). We used the accepted benchmarks of .25 or greater as an effect size that has practical significance (i.e., is educationally meaningful), .5 for an effect size of moderate strength, and .8 as a large effect size (Cohen, 1977).

RESULTS

Table 4 presents the raw data for the number and geometry tests. We computed factorial repeated measures analyses, originally including gender; as no main effects or interactions were significant, we present here only the more parsimonious model.

Table 4
Means and Standard Deviations for Number and Geometry Tests by Site and Group

	Building Blocks					
	Site 1		Site 2		Total	
Test	Pre	Post	Pre	Post	Pre	Post
Number	12.38 (10.94)	36.55 (11.12)	6.13 (6.61)	20.17 (13.29)	9.67 (9.70)	29.46 (14.47)
Geometry	9.53 (2.31)	17.69 (2.64)	7.53 (1.86)	12.87 (3.64)	8.79 (2.26)	15.91 (3.81)

	Comparison					
	Site 1		Site 2		Total	
Test	Pre	Post	Pre	Post	Pre	Post
Number	14.07 (9.39)	26.86 (8.64)	3.17 (2.72)	9.56 (9.34)	8.44 (8.69)	17.93 (12.48)
Geometry	9.56 (1.48)	12.12 (2.10)	7.37 (1.40)	8.62 (3.76)	8.63 (1.89)	10.64 (3.35)

Note. These were the data used for the factorial analyses, so they represent data on those children who took all subtests at both the pretest and posttest.

Table 5

Means and Standard Deviations for Number and Geometry Subtests by Treatment Group

	Building Blocks		Comparison		
Subtest	Pre	Post	Pre	Post	Maximum
		Number			
Verbal Counting	1.13 (1.10)	2.88 (1.51)	0.84 (.93)	1.78 (1.39)	6
Object Counting	5.53 (4.71)	10.97 (4.20)	4.66 (3.89)	8.16 (4.71)	16
Comparing	1.10 (0.99)	2.13 (0.90)	0.89 (0.85)	1.58 (0.96)	5
Numerals	0.60 (1.59)	3.90 (1.86)	0.32 (1.16)	2.48 (2.38)	5
Sequencing	0.07 (0.25)	1.20 (1.24)	0.05 (0.23)	0.39 (0.72)	3
Subitizing	0.18 (0.35)	2.81 (2.63)	0.23 (0.72)	1.00 (1.27)	10
Adding/ Subtracting	0.93 (1.68)	4.20 (2.80)	0.68 (1.38)	2.23 (2.57)	12
Composing	0.13 (0.51)	1.37 (2.31)	0.16 (0.72)	0.32 (0.91)	15
Total	9.67 (9.70)	29.46 (17.95)	7.83 (8.28)	17.93 (12.48)	72
		Geometry, Measurement, Patterning			
Shape Identification	5.42 (0.92)	7.34 (1.16)	5.66 (0.93)	5.89 (1.42)	10
Composition	1.07 (1.10)	4.47 (1.92)	1.23 (1.36)	2.01 (1.65)	11
Congruence	1.02 (0.38)	1.32 (0.35)	1.05 (0.39)	1.20 (0.52)	2
Construction	0.09 (0.23)	0.61 (0.56)	0.09 (0.29)	0.38 (0.48)	2
Orientation	0.15 (0.21)	0.31 (0.28)	0.08 (0.15)	0.08 (0.14)	1
Turns	0.38 (0.49)	0.41 (0.50)	0.21 (0.42)	0.27 (0.45)	1
Measurement	0.10 (0.31)	0.19 (0.40)	0.08 (0.18)	0.08 (0.23)	1
Patterning	0.50 (0.55)	1.26 (0.78)	0.23 (0.42)	0.73 (0.67)	2
Total	8.79 (2.26)	15.91 (3.81)	8.63 (1.89)	10.64 (3.35)	30

Note.

Table 6—*Continued*

	Experimental		Comparison	
	Pre	Post	Pre	Post

description of the different abilities of the two groups. For the first object counting item, about 66% of both experimental and comparison children could provide a verbal response at pretest. At posttest, 100% of experimental children did so, however 16% of the comparison children reproduced the set but could not give the verbal responses and 23% gave no response. On a number comparison item, experimental children increased their use of a counting strategy more than comparison children, over half of whom did not respond. On items in which children counted scrambled arrangements of objects, the experimental group increased their use of strategies more than the comparison group, especially systematic strategies such as progressing top to bottom, left to right. On the arithmetic items, more children used objects, and fewer used verbal strategies (a small minority on the comparison item, "how many dogs wouldn't get a bone?").

Examining the addition learning trajectory reveals the curriculum's positive effects in more detail. Increases in percentage correct from pretest to posttest for four illustrative items were as follows: 2 + 1 (increase of 37 for experimental vs. 23 for comparison), 3 + 2 (47 vs. 13) 5 + 3 (23 vs. 16), 6 – 4 (how many dogs wouldn't get a bone?—23 vs. 10). The curriculum follows the learning trajectory described in Figure 1. On average, children worked on *Nonverbal Addition* activities 4 times, half on computer (see Figure 1) and half off computer. Teachers modeled nonverbal strategies but also encouraged post hoc verbal reflection. Children worked on *Small Number Addition* activities 6 times, 2 on and 4 off computer (Figure 1). Teachers focused on the meaning of addition as combining two disjoint sets, expressed informally. Children worked on *Find Result* activities 6 times, 2 on and 2 off computer. Use of a child's invented counting strategies to solve join, result unknown problems was emphasized. Finally, children worked on *Find Change* problems 2 times, half on and half off computer. Both on- and off-computer activities emphasized counting on from a given number. The results of these activities is shown in the greater than double increase in correctness by the experimental group, as well as their greater use of solution strategies overall and greater use of more sophisticated strategies, such as verbal counting strategies, for most tasks. For example, on 5 + 3, 33% of the experimental children, compared to 19% of the comparison children, used objects, and 23% of the experimental children, compared to less than 7% of the comparison children, used verbal counting strategies. These results are particularly striking when considering that such tasks are normally part of the first grade curriculum.

Table 6 shows four codes describing children's strategies on a shape composition task. By posttest, experimental children were far more likely to combine shapes without leaving gaps; turn shapes into correct orientation prior to placing them on the puzzle; search for a correct shape; and solve the puzzle immediately, systematically, and confidently. This increased use of more sophisticated shape composition strategies suggests the development of mental imagery.

This development of more sophisticated strategies in the experimental group, along with the large relative gains on the subtest score, more than four times as large as those made by the comparison group (Table 5), substantiate the curriculum's positive effect on geometric composition. The curriculum engages children in several activities to develop this competence, including creating free-form pictures with a variety of shape sets, such as pattern blocks and tangrams, and solving outline puzzles with those same shape sets. Informal work with three-piece foam puzzles and clay cutouts were conducted for several weeks during mid fall. In April, the outline puzzles, which provided the most guidance along the learning trajectories, were introduced. Most children in the present classrooms worked about 2 days on the puzzles designed for children at the *Pre-Composer* level (see the third column

of the *Building Blocks* materials, with achievement gains near or approximately equal to those recorded for individual tutoring. This provides support for the efficacy of curricula built on comprehensive research-based principles. The *Building Blocks* materials include research-based computer tools that stand at the base, providing computer analogs to critical mathematical ideas and processes. These are used, or implemented, with activities that guide children through research-based

Ramey & Ramey, 1998). It extends this research by suggesting that a comprehensive mathematics curriculum following NCTM's standards (2000) can increase knowledge of multiple essential mathematical concepts and skills (beyond number).

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