Identifying Student Misconceptions with Formative Assessment Math Probes

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Introduction

In today’s classrooms, teachers are called upon to gather and use evidence of student thinking in a timely, formative way to implement and/or differentiate instruction that improves mathematics learning for all students. An important part of that process is “to identify and address potential learning gaps and misconceptions when it matters most to students, which is during instruction, before errors or faulty reasoning becomes consolidated and more difficult to remediate.” (National Council of Teachers of Mathematics, 2014, p.53).

Powerful tools called Formative Assessment Math Probes are designed to support teachers in identifying student misconceptions. These math probes allow teachers to make sound instructional choices that are targeted at specific mathematics concepts and responsive to the needs of certain groups of students.

The Challenge of Misconceptions

Helping all students build understanding in mathematics is an important and challenging goal. Important steps in achieving this goal include gaining an awareness of student difficulties and the sources of those difficulties and designing instruction to diminish them (Yetkin, 2003).

Misunderstandings are a common difficulty and a normal part of learning mathematics. Many misunderstandings are overgeneralizations, that is, information extended or applied to another context in an inappropriate way. The following examples from the Common Core Progression Documents (http://ime.math.arizona.edu/progressions/) illustrate overgeneralizations:

Focus on Key Words: The language of comparisons can be difficult. For example, “Julie has three more apples than Lucy” means both that Julie has more apples and that the difference is three. Many students “hear” the part of the sentence about who has more, but they may not initially hear the part about how many more. Another language issue is that the comparison statement can be made in either of two related ways, using “more” or “less” (K–5, Counting and Cardinality and Operations and Algebraic Thinking Progression).

Example response of a student with this misconception:

Sam has 7 apples, and Jack has 11 apples. How many more apples does Jack have?

“18 apples, because more means add.”

Place Value: The decimal point is used to signify the location of the ones place, but its location may suggest there should be a “one-ths” place to its right to create symmetry around the decimal point (K-5, Number and Operations in Base Ten Progression).

Example response of a student with this misconception:

What is the value of the 4 in the number 5.647?

“The four means 4 tenths in this number because it goes ones, tens, hundreds on both sides but with a ths instead of a just an s.”

It is important to note that a misconception begins with flawed conceptual understanding and can lead to errors in mathematical application. Mathematical errors can and do occur because of misconceptions, but this is not true of all mathematical errors. For example, a copying mistake or simple calculation error within a large, multiple-step process are errors that are not caused by misconceptions.

The topic of rational numbers further illustrates the challenges of mathematical misconceptions. Many students' difficulties and errors with rational number concepts derive from common conceptual flaws. These flaws can stem from the inherent properties of rational numbers and the transition from learning about and working with the whole number system. While whole number relationships are based on additive properties, rational numbers have relationships based on multiplicative relations. Moreover, rational numbers can be expressed in many different forms (e.g., common fractions and decimal fractions), they use a new system of symbols, and they can be designated by an infinite number of equivalent representations.

When rational numbers are introduced, two numbers that students previously understood as functioning independently now compose a new number (e.g. a common fraction) that has a distinct value. To add further complication, a single rational number can be representative of several distinct conceptual meanings. These distinct conceptual meanings are referred to as "sub-constructs" of a rational number. These sub-constructs include: part-whole relation (4 of 5 equal shares); quotient interpretation (implied division in which 2 submarine sandwiches are divided for 3 boys); measure (fixed quantity on a number line); ratio (5 girls to 6 boys); and multiplicative operator (scaling: reduce or enlarge) (NRC, 2005).

Many students do not understand the meaning of the fraction symbol, and instead focus on either the numerators or denominators when ordering or comparing common fractions. When comparing a fraction such as $\frac{5}{7}$ to $\frac{4}{9}$, they will compare 7 and 9. Students with this misconception may conclude that since 9 is greater than 7, then $\frac{4}{9}$ is greater than $\frac{5}{7}$. Students also hold common misconceptions about the addition of rational numbers. They may add the two fractions $\frac{1}{5}$ and $\frac{1}{4}$ and think the solution is $\frac{4}{9}$ (because they add the numerators together and they add the denominators together).

Often students have difficulty with the fact that the two numbers (numerator and denominator) composing a common fraction are related through multiplication and division, not addition. This may create problems when comparing fractions, because students look at the difference between numerator and denominator to come to an inaccurate conclusion. When looking at $\frac{7}{8}$ and $\frac{5}{6}$, they may see that in each case the difference between numerator and denominator is one, wrongly concluding that these two fractions are equal. In each of these examples, the difficulties described stem from a flaw in the student's conceptual understanding of rational numbers.

It is impossible to teach in a way that avoids creating any misconceptions, so we should accept that students will make some incorrect generalizations that will remain hidden unless the teacher makes specific efforts to uncover them (Askew & Wiliam, 1995). A teacher’s role is to minimize the chances of students harboring misconceptions by acknowledging potential difficulties, using assessments to elicit misconceptions, and implementing instruction to help students build conceptual understanding of the mathematics.

The Power of Formative Assessment Math Probes

A Formative Assessment Math Probe is a short, highly-focused, quick-to-administer diagnostic assessment designed to pinpoint specific misconceptions students may have about a mathematical concept. Math probes have been developed, field-tested, and implemented by teachers for more than 10 years (Keeley & Rose-Tobey, 2006, 2011, 2017). Along with collections created for specific grants, published sets of math probes (like those in the series *Uncovering Student Thinking*) offer specific, grade-level assessments that promote deep learning (Rose, Minton, & Arline, 2007; Rose & Arline, 2009; Rose & Minton 2010; Rose-Tobey & Arline, 2014; Rose-Tobey & Fagan, 2013, 2014).

Math probes from all resources have important common characteristics. Each math probe typically includes three to six items, and each item requires a two-part response from the student: a selected response and a written explanation using words and/or pictures. Together, this combination helps to reveal underlying patterns in incorrect answers and will show whether correct selected responses are supported by strong or by faulty reasoning. Multiple items targeting a specific topic provide important insights into why a student may be having difficulty.

Following are two examples of math probes:

Multiplication Equations

A) Circle the number that belongs in the box: 2 4 6 24	Explain how you got your answer.
B) Circle the number that belongs in the box: 3 6 18 54	Explain how you got your answer.
C) ( ) ( ) Circle the number that belongs in the box: 3 4 16 24 48	Explain how you got your answer.
D) ( ) ( ) Circle the number that belongs in the box 4 8 17 18 72	Explain how you got your answer.

Math Probes and Formative Assessment

A 1998 review of more than 250 articles related to formative assessment highlights the importance of formative assessment in learning and teaching (Black and Williams,1998). Since then, researchers and practitioners have investigated the topic more deeply and clarified the attributes of formative assessment, resulting in an expanded definition. Formative assessments

> "Using math probes has helped me to analyze student work in a different way. By looking for patterns in thinking of both correct and incorrect selected responses, I am often surprised in both directions—what they know and what they don't know."
>
> – Grade 2 Teacher

Math Probes and Responsive Action

While diagnostic assessment involves learning about student thinking, formative assessment involves using what you've learned about students' understandings to design and implement instructional experiences. Math probes are considered diagnostic until a teacher acts on the data to move student learning forward. When teachers are clear about the conceptual understanding they are working to build, and they have analyzed the math probe results, they are well situated for the next step in the process: moving from diagnostic to formative assessment by providing targeted learning activities.

> "The phrase 'to teach without causing misconceptions is impossible' really resonates with me. Using math probes helps me learn how students have interpreted my instruction and the mathematics activities I have asked them to engage in. I can then more easily pinpoint additional experiences that will support their learning."
>
> – Algebra II Teacher

Consider the following classroom vignette:

> To get a sense of what my students understood from our initial lessons on fraction addition, I ga probe on estimating fraction sums. In reviewing my students' responses to the math probe, I no following:
>
> > A few students approached the problem by estimating percent equivalents and adding thos determine whether the sum was more or less than the benchmark. They used this strategy e and many used this strategy across each of the problems on the math probe.
> >
> > A handful of students misapplied whole-number thinking, such as adding the numerators an the denominators to find the sum of the fractions. These students did not use estimation at a
> >
> > Another group of students did not estimate but rather correctly applied the addition algorithm the sum of the fractions. While these students got correct answers, their lack of estimation r questions for me about their ability to reason about the size of the fractions and the operatio addition.
> >
> > A few students used varied estimation strategies to choose the correct selected response, a supported their selection with solid reasoning.
>
> By looking for patterns in understandings and misunderstandings, I was better able to pinpoint th problems students were having, allowing me to explore and discuss them in the upcoming lesso Fagan, 2014, p.186-187).

When and How to Use Math Probes

Teachers need to be clear with students about the purpose of the math probe and how they will

References

Askew, M., & Wiliam, D. (1995). *Recent research in mathematics education 5–16.* London, UK: HMSO.

Black, P., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy & Practice*, *5*(1), 7–73.

Common Core Standards Writing Team (CCSWT). (2011). Progressions documents for the common core math standards, Draft 6–8, Progression on expressions and equations. Retrieved from http://ime.math.arizona.edu/progressions/#products

Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards in Mathematics (draft): K–5, number and operations in base ten.* Retrieved from http://ime.math.arizona.edu/progressions/#products

Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards in Mathematics (draft): K–5, number and operations—fractions.* Retrieved from http://ime.math.arizona.edu/progressions/#products

Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards in Mathematics (draft): K–5, operations and algebraic thinking.* Retrieved from http://ime.math.arizona.edu/progressions/#products

Common Core Standards Writing Team. (2011a). *Progressions for the Common Core State Standards i Mathematics (draft): K, counting and cardinality.* Retrieved from http://ime.math.arizona.edu/progressions/#products

Common Core Standards Writing Team. (2013). *Progressions documents for the common core math standards, draft high school, progression on functions.* Retrieved from http://ime.math.arizona.edu/progressions/#products

Council of Chief State School Officers (CCSSO). (2008). *Attributes of effective formative assessment.* Retrieved from http://www.ccsso.org/

Creighton, S.J., Rose Tobey, C., Karnowski, E., Fagan, E. (2015). *Bringing Students into the Formative Assessment Equation: Tools and Resources for Math in the Middle Grades.* Thousand Oaks, CA: Corwin.

Heritage, M. (2010). *Formative assessment: Making it happen in the classroom.* Thousand Oaks, CA: Corwin.

Fagan, E., Rose Tobey, C. & Brodesky, A. (2016). *Targeting Instruction with Formative Assessment Math Probes.* Teaching Children Mathematics. Vol. 23, No 3

Keeley, P. and Rose, C (2006). *Mathematics Curriculum: Bridging the Gap Between Standards and Prac* Thousand Oaks, CA: Corwin Press.

References (continued)

Keeley, P., & Tobey, C. (2011). Mathematics formative assessment: 75 practical strategies for linking assessment, instruction, and learning. Thousand Oaks, CA: Corwin.

Keeley, P., & Tobey, C. (2017). Mathematics formative assessment: 50 more practical strategies for linki assessment, instruction, and learning. Volume 2. Thousand Oaks, CA: Corwin.

National Council of Teachers of Mathematics. 2014. Principles to Actions: Ensuring Mathematical Succe for All. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (2005). How students learn: Mathematics in the classroom. Washington, DC: