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For many adults, and even many of today's students, math class is a place where a teacher talks and students listen, responding only when called on to answer a question.

Much of this shift is attributable to differences in the nature of discourse in the classroom: Who talks, and what do they talk about? Who asks questions, and what kinds of questions do they ask? What are students thinking, and how do they back up their ideas? Such questions provide hints about the kind of classroom discourse that best supports student learning and how we can create an environment in which rich discourse can take place.

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There is growing consensus that students need frequent and regular opportunities to express their thinking and interact with others about their ideas. One way to ensure that this happens is by rethinking a teaching model used extensively in mathematics classrooms over the years.

Many educators were taught to learned to present a rule, procedure, or concept accompanied by examples, then work through guided practice with students before asking them to apply their new knowledge to one or more problems. This process is sometimes characterized as 'I-We-You'— (the teacher) explain, (students and teacher together) work examples through guided practice, (students) practice on your own.

Today, we know that students can benefit from wrestling with challenging problems. lays a foundation for thinking about mathematical ideas and developing their reasoning (Pasquale, 2015; Warshauer, 2014; NCTM, 2014; Seeley, 2015). If we start with a mathematically rich task or problem, we can turn the traditional 'I-We-You' model upside down to utilize a 'You-We-I' structure for teaching (Seeley, 2014), starting with a rich problem students don't already know how to solve or a task they haven't previously explored. In this kind of upside-down model, (students) work on a problem, (students with the teacher facilitating and probing) talk about what students think and their possible approaches to the problem, asking questions of each other and revealing their understanding, and (the teacher) ensure that the resulting discourse culminates with students learning the intended mathematics of the lesson.

Providing students an opportunity to grapple with challenging ideas that they don't already know, or problems that go beyond what they may have seen before, gives them something to communicate about. That communication, via written or spoken words or other representations, can clarify their thinking and reveal understandings and misunderstandings. Sometimes, in the process of talking through a solution or idea, a student may catch and often correct an error she or he has made. When a student develops a strategy or approach based on sound mathematical understanding, communicating what he or she did can increases the likelihood that the student will be able to use or adapt that strategy in future situations.

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While all students can benefit from opportunities to talk about their thinking and reasoning, it may be even more important for students who are not strong in English to engage in discourse that goes beyond superficial language.

Dr. Susie Håkansson (2017) suggests that it is important for English language learners to engage in mathematical discourse focused both on receptive language functions (listening and reading) as well as productive language functions (speaking and writing). Having students who are not proficient in English use more language in the mathematics classroom, rather than less, may be counterintuitive. A teacher's first inclination may be to give such students mathematics involving fewer words, focusing on numbers or symbols. But to do so would be to deny these students the opportunity to engage in rich problems and would put them at a disadvantage, potentially causing them to fall farther behind their English-proficient classmates.

Language experts remind us that teachers should learn to amplify and enrich, rather than simplify classroom language so that students have more opportunities to learn the intended concepts (Walqui & van Lier, 2010). Dr. Håkansson reiterates this call for amplification and suggests that it is important for teachers to go beyond simply focusing on words, phrases, vocabulary, and definitions to help students learn to use a full range of discourse tools to express and deepen their emerging mathematical understanding. She notes the importance of students using a variety of representations, including graphical and pictorial representations, as important components in their mathematical discourse. Representational tools can greatly aid all students—both those who are learning English and those who are proficient in English—in developing their mathematical language, as well as deepening their understanding of mathematical concepts, as described in NCTM's representation standard (2000).

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The most productive discourse involves students thinking about, reasoning about, and making sense of mathematical ideas, problems, concepts, or even procedures. Engaging in mathematically rich tasks that challenge students somewhat beyond what they already know can set the stage for rich discourse around mathematical ideas.

If students are continually expected to share their thinking and listen to the thinking of others, they will become more accustomed to doing so. Over time, they become increasingly more sophisticated in their level of language and their depth of mathematical understanding (Chapin, O'Connor, & Anderson, 2013; Boaler, 2015). They get used to the notion that it's ok to struggle a bit over a challenging problem, even to make a mistake, knowing that the discussion that evolves from their struggle is likely to lead to better understanding.

NCTM has suggested that the most effective student learning arises when tasks are carefully selected to elicit thinking and when the teacher poses questions that support the development of reasoning (NCTM, 1991; NCTM, 2014). However, they also note that tasks with a high cognitive demand may be challenging to teach and that teachers sometimes have a tendency to lower the level of the task during instruction (NCTM, 2014).

Some video examples of tasks that can engage students and elicit thinking are listed at the end of this paper. They include:

- Using a homemade video of a provocative situation to generate students' questions and solutions (a mysterious hand reaches up to grab some cookies)
- Posing a fairly straightforward word problem (asking students to determine how many cups

Teachers can learn to ask questions that help students move through increasingly productive levels of mathematical discourse. To do so, teachers should focus on questions that stress mathematical thinking rather than just answers ("Why do you think so?" "Can you explain what you mean when you say that there will always be a leftover?") and questions that probe for understanding ("How did you decide to multiply by 10?" "Where does the 7 in the equation show up in your graph?"). It is important to pay attention to both the quality of the discourse and also the content. The ways in which students share their ideas is important, but likewise discussion needs to be guided to address productive mathematical ideas (Bransford and Donovan, 2005).

If we want students to be willing to express their thinking and take the risk of sharing their ideas in front of their peers, we need classrooms where everyone's contribution is valued and where it's safe to take such risks. Creating such an environment calls for initiating community discussions about respecting each other in the classroom and establishing norms for how conversation and questioning will take place. Further, students need to accept that mistakes are likely to happen, and, in fact, that mistakes serve as a stimulus for rich discussions and powerful learning (Seeley, 2016). Jo Boaler (2015) reminds us that we learn more—and our brain grows more—when we make a mistake than when we get a right answer.

One particular challenge for teachers is how to balance the goal of accuracy in mathematics with helping students see the learning potential from mistakes. It is important to talk with students about both of these dimensions of our mathematics classes. There's nothing wrong with striving for accuracy (within humane limits), while at the same time embracing mistakes. In fact, some mathematics problems, particularly in the area of mathematical modeling, may not yield a single, clear right answer. Too many adults today carry the belief that they're just not 'math people' because of the fact that they have made mistakes in mathematics. We cannot underestimate the negative long-term effect of subjecting students to failure messages that interfere with their learning and their interest in learning mathematics. Nevertheless, getting to correct answers is a reasonable goal in mathematics. After all, mathematics is a discipline grounded in patterns and predictability, generating reliable and consistent answers to many problems.

H C O I C D -O C ?

Many excellent teacher education and professional development programs help teachers learn skills in organizing classrooms rich in the kind of student discourse that leads to learning. We can recognize such excellent programs and expand other programs to support this kind of teacher learning.

Teacher evaluation and reward systems can be redesigned where necessary to value and encourage the kind of mathematics teaching expertise that pushes student-thinking while building a strong and deep foundation of conceptual and procedural knowledge.

Orchestrating rich, productive discourse in a mathematics classroom is certainly among the most important elements in mathematics teaching today. Classroom discourse about mathematics can be a tool for equity, a vehicle for developing reasoning, and an engine for lasting learning.

References (con in ed)

Kilpatrick, Jeremy, Swafford, Jane, & Findell, Brad (Eds.). (2001). *A* . Washington, DC: National Academy Press.

Lampert, Magdalene. (1989). . In J. Brophy (Ed.), Advances in Research on Teaching (pp. 223-264). Greenwich, CT: JAI Press.

National Council of Teachers of Mathematics. (1991). . Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). . Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2014). *A* *A* . Reston, VA: National Council of Teachers of Mathematics.

Pasquale, Marian. (2015). . . Boston: Education Development Center.

Seeley, Cathy. (2014). - . *A* , , (pp. 88-97). Sausalito, CA: Math Solutions.

Seeley, Cathy. (2015). . *A* , , (Second ed.). Sausalito, CA: Math Solutions.

Seeley, Cathy. (2016). . Alexandria, VA: ASCD.

Smith, Margaret Schwan, & Stein, Mary Kay. (2011). 5 . Reston, VA: National Council of Teachers of Mathematics.

Walqui, Aida, & van Lier, Leo. (2010). *A* *A* *A* . San Francisco, CA: WestEd.

Warshauer, Hiroko Kawaguchi. (2014). ***I*** .