for length does not mean they will automatically realize that units are also iterated for area or volume/capacity. So, the concepts that run in parallel for length, area, and volume/capacity must be made explicit and repeated for each dimension; they must also be connected with one another.

Research indicates that even very young children typically have the ability to recognize objects as larger or smaller without attending to a particular dimension. Before age five, they are able to recognize length as a dimension separate from other measurable attributes of objects such as area, volume/capacity, or weight (Barrett, et al., 2011; Nguyen, 2010). At this point, they begin to make holistic comparisons of lengths based on how things look, without making direct comparisons (Barrett, et al., 2011; Battista, 2006; Clements, et al., 2006; Nguyen, 2010; Smith, et al., 2008). Next, they start to refine vague visual comparisons of lengths into direct comparisons (Barrett, et al., 2011; Battista, 2006; Clements, et al., 2006; Nguyen, 2010; Smith, et al., 2008). Not only will they look at two paths and say that one looks a little longer than the other, but they may also try to line up the ends of the paths (Barrett, et al., 2011; Battista, 2006; Clements, et al., 2006; Nguyen, 2010; Smith, et al., 2008). These qualitative comparisons of length seem to be an important early step toward understanding quantitative measurement. Being able to say that one object is shorter than another allows children to then use the shorter object as a unit for measuring the longer one. (Clements, et al., 2006; Smith, et al., 2008)

Just as holistic, visual comparisons of length serve as a precursor to direct comparisons of length, counting to find length—regardless of unit iteration—serves as a precursor for quantitative measurement that involves correct unit iteration (Clements, et al., 2006). For example, children may count as they move their fingers along a path or count unevenly spaced, differently sized beads on a string in an attempt to measure the lengths of the path and string (Clements, et al., 2006).

enumerate units properly (without gaps, overlaps, etc.) (Barrett, et al., 2011; Battista, 2006; Clements, et al., 2006; Smith, et al., 2008). Next children work toward correct unit iteration, when they can position unit lengths end-to-end along an object and count them to arrive at the measure (Barrett, et al., 2011; Battista, 2006; Clements, et al., 2006; Smith, et al., 2008).

able to define the units they use (standard and non-standard) with actual size drawings, scale drawings, or verbal statements (Smith, et al., 2009).

Understanding that a path is both a set of added lengths and a single length, and that those two lengths have the same measure, is called “integrated conceptual path measuring” (Barrett, et al. 2011). It is developed and practiced by adding iterations of units[3]. Battista shows this in activities where children find perimeters of rectangles by counting and adding lengths of sides (Battista, 2004). This understanding that length is additive allows children to begin adding lengths without iterating (Barrett, et al., 2011; Battista, 2004). In Battista’s research, this entailed children finding the perimeter of a rectangle by adding the lengths of sides. Children who have achieved this understanding can see lengths as continuous and also as numbers that obey

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