During the early planning phase of *Everyday Mathematics 4*, a team of writers worked together to read and summarize the current body of research about fractions. The information that follows is the summary of that research. For more information about the planning phases of *Everyday Mathematics 4*, see the paper "*Everyday Mathematics* and the Writing Process."

Summary of the Examined Research

At the inception of our work in researching the conceptual development of mathematical concepts, we chose to focus our literature searches and reviews on learning trajectories and progressions for each strand. This focus stemmed from information in a document from the Consortium for Policy, Research, and Education that connected current research on learning trajectories with the Common Core State Standards (CCSS) (Daro, et al., 2011). As a result, for the rational number strand, we focused our reading on sources that included at least one component of a learning trajectory as defined by Confrey, et al.:

"A researcher-conjectured, empirically-supported description of the ordered network of experiences a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time" (Confrey, et al., 2008).

For the research summary that follows, we concentrated on those sources that included a sequence of concepts, instructional activities, or both, that covered a significant portion of the development of initial fraction concepts and operations. The narrative does not include a discussion of the research that supports the progressions or trajectories we reference. However, the authors of these sources represent long-standing and respected organizations in the field of

and denominators. (Wilson, 2009; Confrey, et al., 2008; Empson & Levi, 2011; Fosnot & Dolk, 2002; Lamon, 2005; Lamon, 2007; RNP, 2009a; RNP 2009b).

Empson & Levi, Fosnot & Dolk, and, to some extent, Lamon and RNP recommend the development of meaning for fractions (e.g., concept of a whole) and fraction operations through the use of word problems in everyday contexts. In particular, Empson & Levi and Fosnot & Dolk rely on students' drawings and explanations of invented strategies for fraction representations. Wilson describes using counters for sharing collections and folding paper rectangles and circles to partition regions (Wilson, 2009; Empson & Levi, 2011; Fosnot & Dolk, 2002; Lamon, 2005; RNP, 2009a; RNP, 2009b).

The general progression for developing fraction concepts through equal sharing and partitioning begins with fair shares of collections and regions for two people (halving and doubling) with no remainders, moves to equal sharing in which students must reason about remainders that produce denominators of 2 and 4, extends to small odd numbers, and continues to larger denominators. For example, Wilson includes a progression with eight levels that culminates in solving problems that involve m objects shared among p people ($m<p$ for proper fractions and $m>p$ for improper fractions and mixed numbers). The values for p (the denominators) begin with 2 and 4 and progress to include odd numbers and larger denominators (Wilson, 2009; Myers, et al., 2009). For each level Wilson delineates a within-level framework for activities and language that grow in sophistication (Wilson, 2009):

1) Methods: Solve the task (How could you share?)
2) Multiple methods: Solve the task in multiple ways. (Is there another way to share?)
3) Justification: Justify solutions. (How do you

In general, the researchers recommend delaying the use of fraction symbols and to allow for the development of conceptual knowledge. This delay helps students avoid errors from attending to superficial features of fraction notation (Empson and Levi, 2011) and from misapplying whole number ideas as they begin work with fractions. Students use language (such as halves, fourths,

Rational Number Project. (2009b). Rational number project: Fraction operations and initial decimal ideas. (Companion model to RNP: Initial fraction ideas.) Retrieved at http://www.cehd.umn.edu/rationalnumberproject/rnp2/rnp2.pdf.

Wilson, P. H. (2009). Understanding the Effects of a Learning Trajectory for Equipartitioning in Classrooms: A Mixed Methods Investigation (dissertation). North Carolina State University, Raleigh, NC.