ENRICHING ADDITION AND SUBTRACTION FACT MASTERY THROUGH Gam

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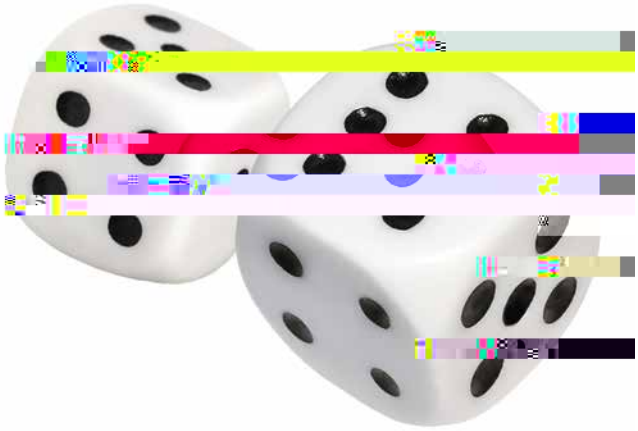
Becoming uen i h ba ic fac i de elopmen al. U e game infu ed i h a focu on acqui i ion of ra egie o help uden in he earl grade progre o compu a ional uenc .

The learning of basic facts single-digit combinations for addition, subtraction, multiplication, and division has long been a focus of elementar school mathematics. Man of us remember completing endless worksheets, timed tests, and ash card drills as we attempted to master our basic facts as children. However, research over the past thirt ears, recommendations from *A* (NCTM 2014) on effective mathematics teaching practices, and goals for students outlined in the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010), suggest a ver different approach that has the promise of greater student engagement and success. Ke to an effective approach to teaching basic facts is an understanding of the phases through which students progress as the learn their basic facts and a reali ation of how practice can be used to help students master their facts (Van de Walle, Karp, and Ba -Williams 2013). We illustrate how both of these aspects can be



Understanding the phases of learning basic facts

Fluenc with basic facts can be de ned as the ef cient, appropriate, and e ible application of calculation skills and is an essential aspect of mathematical pro cienc (Barood 2006, p. 22). Barood describes the following three phases through which students must progress as the develop master with a particular group of facts:

1. Modeling and/or counting all or counting on to nd the answer; for e ample, using ngers to help keep track of their counts to solve 5 + 7 = ?
2. Deriving answers using reasoning strategies based on known facts, such as solving 5 + 7 b thinking, Five plus ve equals ten, and two more will make twelve.
3. Master or ef cient production of answers. For e ample, when asked, What is 5 + 7? a child might call out, Twelve, and e plain, I just knew it.

Traditional approaches to learning facts gener- all ignore the second phase and move children quickl from beginning conceptual e peri- ences with addition and subtraction to rote memori ation of facts via drill, ash cards, and timed testing. Although pushing students from phase 1 to phase 3 is possible, when students have not developed reasoning strategies to ef cientl nd a basic fact, the are unable to regenerate the answer when the forget what the have memori ed. In contrast, a uenc approach allows the third phase to develop out of meaningful e periences with phase 2, as children create, share, evaluate, and practice ef cient strategies for nding unknown facts from facts the have alread mastered. The result is not just a much richer mathematical e perience, but one necessar to establish pro- cedural uenc (Barood 2006; Brownell and Cha al 1935; Carpenter and Moser 1984; Henr and Brown 2008) and higher student achieve- ment (Thornton 1978, 1990; Steinberg 1985).

CCSSM e pectations for K grade 3 use the term in a wa similar to phase 2 from Barood 's framework and as something from automaticit . For e ample, in grade 1 the CCSSM e pectation is as follows:

> Add and subtract within 20, for addition and subtraction within 10. such as counting on; mak- ing ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. (CCSSI 2010, 1.OA.C.6)

In grade 2, children progress toward master . Standard 2.OA.B.2 states, add and subtract within 20 using mental strategies. B end of grade 2, all sums of two one-digit numbers. Note the distinction between the word and the phrase within the standard, which, along with the details on strategies given in 1.OA.C.6, strongl suggests that CCSSM recogni es the importance of allowing students to progress through phase 2 before e pecting automaticit with their facts (phase 3).

How can teachers ensure that their stu- dents acquire the strategies needed to master phase 2? E plicitl teaching strategies does mean teaching a speci c strateg and then asking students to use it. Such an approach removes the reasoning from the reasoning strateg and instead adds to what a student is being asked to memori e. In fact, students in classrooms with a heav emphasis on just memori ing basic fact strategies have been shown to have lower number sense than students whose teachers do not rel heavil on memori ing strategies (Henr and Brown

strategies that help them get to the solution without counting. Gravemeijer and van Galen (2003) call this approach [illegible] because not all the reasoning strategies will be used b students without some guidance. Finall , this process takes time, often more time than we think it will take. For e ample, it can take between two and four lessons before most students reall internali e the reasoning strategies discussed in class (Steinberg 1985). Opportunities for students to practice choosing strategies can occur in a wide variet of settings, but it is critical that practice is purposefull used. That is, a ke element of developing fact uenc is meaningful practice.

Meaningful practice

As students progress through the CCSSM e pectations for K grade 2, it is necessar for teachers to provide opportunities for meaningful practice that both engages and respects students' developmental levels. Meaningful practice of facts can come in man forms, including using stor problems, ten frames, and games. Several research-based elementar curricula (for e ample, Ever da Mathematics 4 [Bell et al., forthcoming] and Investigations in Number, Data, and Space [Russell et al. 2008]) rel heavil on the use of games to provide engaging practice for their students, often replacing routine pencil-and-paper tasks. Children's enthusiasm for such games cannot be overestimated; ver often when working with rst and second graders, we have found students reluctant to stop pla ing when time is up! Games ma be designed for either targeted practice (on a particular group of facts) or general practice (all facts for a particular operation), and the strategic use of such games can help move students along the different phases of uenc . As children pla games, ou can observe and interview individuals to monitor their progress through the phases (Kling and Ba -Williams 2014). These forms of assessment provide better data, while replacing the need for timed tests, which potentiall have a negative impact on children (Boaler 2014). In the sections that follow, we highlight games that are motivating and useful for helping oung children progress along the three phases toward meeting the CCSSM e pectations for master of their addition and subtraction facts.

FIGURE 1

Below are brief descriptions for the High Roller game and the Roll and Total game (Bell et al., forthcoming).

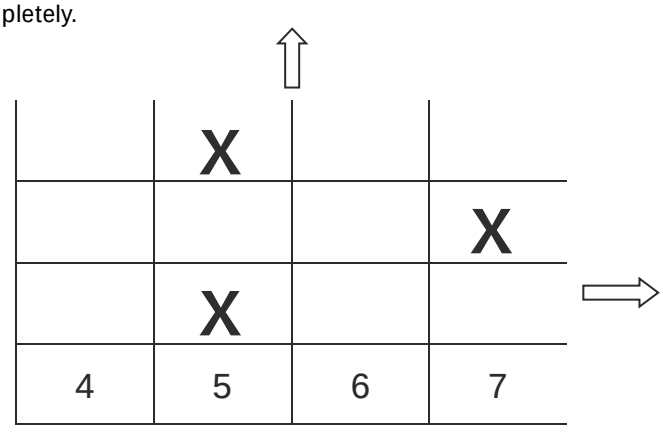
(a) H R

Players take turns, each time rolling two regular dot dice. After the first player rolls his two dice, he determines which one has the greater number (the high roller) and keeps that die as it is while rolling the other die a second time. He then counts on from the first die to get the sum of the two dice and records the sum. The second player repeats the process, and play continues as time allows.

(b) R a d T a

Students begin with a table with 11 columns on it, each labeled with a number 4–14. Each column has at least eight rows. (See below for a partial table.)

Working with a partner or alone, students roll two dice—one is a regular dot die, the other is a die with the numerals 3–8 recorded on it. Students then count on from the numeral die, using dots from the regular die, as needed, to find the sum, which is recorded in the appropriate column. Play continues until one column is filled completely.



Moving within phase 1 toward phase 2

A major developmental milestone for children acquiring uenc with their basic facts is moving from counting all to counting on. This t picall occurs in kindergarten or earl in rst grade, and it is the rst strateg listed in the grade 1 standard 1.OA.C.6 (CCSSI 2010). Although it is necessar to respect the fact that this is a developmental milestone for children, and as such, cannot be forced, we have found that two games are helpful in nudging students toward counting on. In the High Roller (see **fig. 1a**) game (Bell et al., forthcoming), children begin b rolling two dice and keeping the one with the larger number. The then roll the die with the smaller

amount again and count on from the rst die to nd the total. Since the rst die is alread ed, this game encourages students to count on as opposed to counting all when nding their sums. This skill can also be encouraged with the Roll and Total game (see **fig. 1b**) (Bell et al., forth

FIGURE 2

Below are brief descriptions for the Tens Go Fish and Double It games (Russell et al. 2008).

(b) D . b I

Students begin the game by using a table (see below). The table for the Double It game has nineteen columns, each labeled with a number 2–20. Each column has eight

11 12 18 18 21 B 14 93t(18)Tjdw[\a 61918 53t(18)T(.293u 5bn rla

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

can result from e tensive use of strategies (phase 2) until the student is so familiar with a fact, he just knows it (phase 3). In practice and in theor , mathematicians do not memori e, but come to know and understand particular facts and algorithms that are relevant to their work. Although CCSSM does not e pect students to become automatic with their addition facts until the end of second grade (2.OA.B.2), students will begin operating at phase 3 with certain groups of facts much earlier than that. As discussed above, targeted practice can encourage students to become automatic with their doubles and combinations of ten within rst grade. For the

remaining facts, general practice can provide opportunities for students to appl their strategies frequentl enough to become automatic with all facts. Making student reasoning e plicit during pla is a crucial aspect of making this practice mathematicall meaningful. To keep the focus on strateg use during this general practice, asking questions as ou observe is important:

- What reasoning strateg did ou use to gure out the answer?
- Could ou have used a different strateg ?
- Is there a fact ou know that can help ou with this problem?

Such questions keep students focused on the importance of using reasoning strategies and serve as a rich source of assessment data.

The games Addition Top-It and Subtraction Top-It (Bell et al., forthcoming) serve well for general practice. These games are similar to the card game War, e cept that each pla er ips over two cards and nds the sum (or difference) of the two cards. The pla er with the larger sum (difference) then takes all four cards into his deck, and pla continues b ipping over the ne t sets of cards. Once students have begun to develop some strategies for their basic addition and subtraction facts (midwa through rst grade), these games can begin to serve as a canvas for practicing the t pes of fact strategies suggested b CCSSM 1.OA.C.6. Further, children have opportunities to acquire new strategies from their fellow pla ers if the teacher asks questions to elicit strategies used during pla and follows up with prompts to the rest of the group, such as, Do ou understand how she solved it? Can ou e plain what she said in our own words? Such questioning not onl promotes better content understanding but also further encourages children to engage in the Standards for Mathematical Practice, particularl SMP 3: Construct viable arguments and critique the reasoning of others.

As students approach the end of rst grade, the work of becoming uent with subtraction facts can be eased b encouraging them to relate subtraction to addition. In fact, use of this relationship is e plicit in CCSSM in two different standards (1.OA.C.4 and 1.OA.C.6). The game Salute! (Van de Walle, Karp, and Ba -Williams 2013) is e cellent not onl for mental strateg practice but also for seeing the relationship between addition and subtraction (see **fig. 3** for instructions). Notice that the pla ers are emplo ing the think-addition strateg to solve a subtraction fact. For e ample, if the heard the

When the authors play Salute! they have both players say their respective card before the round ends to ensure that every student gets to solve a problem every round.

Sa . !

T _ b c : to be the first of the players to say what number is on your forehead (therefore winning the pair of cards).

Ma _ a : A deck of cards numbered 0–9. We used cards from the Investigations in Number, Data, and Space series (Russell et al. 2008) because players benefit from the ten-frame illustration of the number below the numerals.

This game is for three students: a leader and two players.

1. The leader shuffles all the cards and places them facedown in a stack.
2. The leader hands one card to each player so that the player cannot see his or her own card.
3. The leader says, "Salute!" Each player places his or her card face-out on his or her forehead (players can now see the other player's card but not their own).
4. The leader says the sum of the two cards.
5. A1.889 wee the otalu4f0 Tc7 Tces

sum was 8 and saw a 5 on the other student's forehead, the would think, Five plus what number equals eight? When we pla Salute! we ask that both pla ers sa their card before the round ends. This game provides opportunities

Communicating with families

Man adults with childhood e periences learning mathematics through rote memori ation

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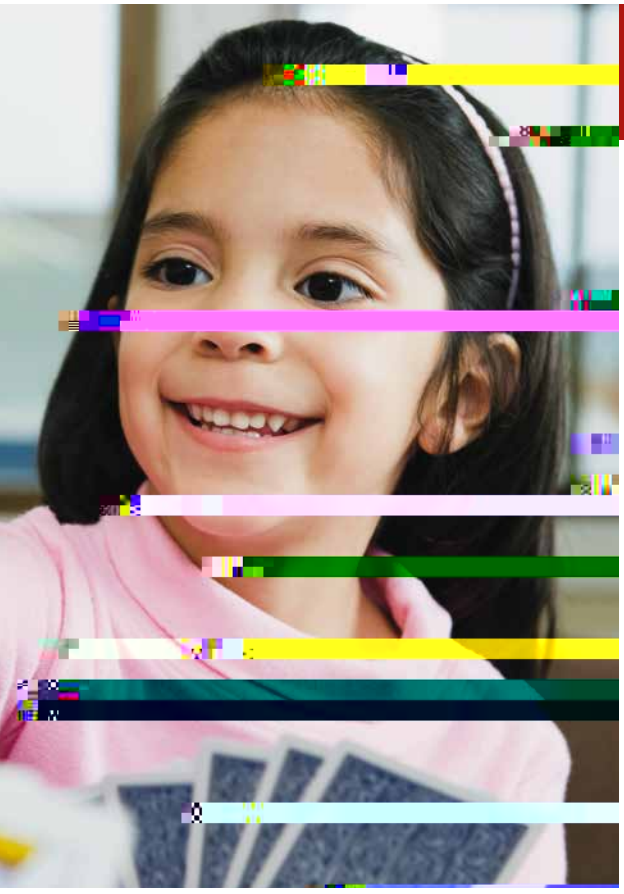

Games engage students in mathematical thinking and help build fact fluency.

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