Unlocking the Gate to Calculus Success: Pre-Calculus for Engineers - An Assertive Approach to Readying Underprepared Students

Prof. Tanya D Ennis, University of Colorado Boulder

Tanya D. Ennis is the current Engineering GoldShirt Program Director at the University of Colorado Boulder's College of Engineering and Applied Science. She received her M.S. in Computer Engineering from the University of Southern California in Los Angeles and her B.S. in Electrical Engineering from Southern University in Baton Rouge, Louisiana. Her career in the telecommunications industry included positions in software and systems engineering and technical project management. Tanya most recently taught mathematics at the Denver School of Science and Technology, the highest performing high school in Denver Public Schools.

Dr. Jacquelyn F. Sullivan, University of Colorado, Boulder
Dr. Beverly Louie, University of Colorado, Boulder

BEVERLY LOUIE is the director for teaching and learning initiatives in the Broadening Opportunities

Unlocking the Gate to Calculus Success: Pre-Calculus for Engineers—An Assertive Approach to Readying Underprepared Students

Abstract

In general, underrepresented engineering students who enter the college underprepared in mathematics lack the basic skills necessary to succeed in calculus. Numerous factors contribute to these issues, including high school mathematics instruction deficiencies such as the absence of qualified teacher resources, poor instruction, and low student interest in higher-level math. The Engineering GoldShirt Program at the College of Engineering and Applied Science at the University of Colorado Boulder is piloting a well-supported new *pre-calculus for engineers* course in the college with the aim to adequately prepare these students for success in calculus, so as to not lose them at the gateway of the engineering education pathway.

In previous semesters in an effort to gain pre-calculus mastery, these GoldShirt students took math courses outside of the college in order to prepare to take the calculus sequence in the engineering curriculum. A subsequent review of these students' results

new first-year class). Recommendations made to the college to expand the pre-calculus for engineers course have not yet been adopted.

This paper examines the performance results of the pre-calculus for engineers course and compares them to

should be successful in calculus I. Yet an overall, high failure rate for calculus I takers persists; more than 35%-40% of students do not pass the course [6,9,10].

Some institutions employ the additional approach of various in-house math assessment tools to guide students into the appropriate math courses. While in the past, students took these assessment tests during orientation and/or the first week of classes, now students submit assessment responses online, enabling them to obtain earlier guidance for enrollment into the most suitable math course (or remediation steps to take). Purdue University's Mathematics Science Inventory assessment relies on self-reporting to determine a student's level of familiarity and confidence in various math topics. The tool has been shown to be effective in helping to place students, as well as providing information to guide students toward academic support resources[11]. At the New Jersey Institute of Technology, assessments to determine algebra proficiency have helped to optimize course placement[12]. By using the Math Inventory in combination with standardized test scores and high school class rank to place students, the University of Pittsburgh reported that 94% of the students enrolled in calculus I performed satisfactorily or better; it also indicated a need for twice as many students to take pre-calculus compared to the prior year[13].

Numerous strategies to boost students' mathematics skill levels have been documented. Some engineering colleges implement math improvement curricula into their summer bridge experiences or summer courses[14,15,16]. Evidence suggests that participation in math refresher activities boosts math assessment test scores, thereby enabling students to avoid remediation courses—calculus preparatory courses such as pre-calculus and algebra—and subsequently gain entry into to t-6.39w -34.syactivities3.48(to)2(r)5(il)-20t9 0 u* [(ex)lel1(imp)2(le)ssh 0 Td

which counted for 10% of the course grade[27,28]. Institutions also use ALEKS as a self-paced learning method to help students improve their knowledge and scores so as to qualify for higher level courses

Figure 1. ALEKS score distribution by demographic for all students directly admitted into the College of Engineering.

It is notable that 20 out of 85 (24%) URM students are not ready for calculus I; likewise, 30 out of 160 (19%) women (including 3 URM women) are not ready for calculus I. When viewed through this lens, 47 out of 242 women and URM students (19%)—students who are key to increasing diversity in the college—are not prepared for success in calculus I.

Engineering GoldShirt Program Overview

The College of Engineering is in the fourth implementation year of a diversifying program called the Engineering GoldShirt Program with the goal to enroll and graduate "next-tier" students with high potential but poor high school academic preparation. Each cohort of 32-36 students was chosen after a day of testing, interviews and observation in team settings. The program's goals are to provide expanded opportunity and a *performance-enhancing "Engineering GoldShirt" year* for motivated high school graduates who are not *yet* fully prepared to succeed in an undergraduate engineering program, as an avenue for the college to increase enrollment and retention of students historically underrepresented in engineering—minorities, women and first-generation college attendees. The Engineering GoldShirt Progrrhigh potent and 2m ave3 tmmu-1(o)-it 0.00ynt

one-third of the college tuition cost, and are expected to complete BS degrees in five years, including the Engineering GoldShirt Program first year. Various elements of the program are designed to build community, ignite excitement about engineering, and prepare students to succeed in engineering. Entering engineering GoldShirt students participate in a two-week summer bridge program to orient them to the challenges of college, building community among their peers, and developing leadership skills through a wide range of activities. During the initial year, students learn in small, cohort-based classes in mathematics, introductory physics, chemistry, writing and critical thinking. These students are placed into appropriate mathematics classes based on an in-depth review of placement exam results and high school transcripts, with the intent to ensure they are prepared to enter or have begun the engineering calculus sequence at the close of their first year. Students who achieve predefined metrics in the first GoldShirt year continue on with the standard curriculum for their engineering major, along with ongoing required participation in community-building and service-learning activities throughout their subsequent four years. This high-touch program is housed with the BOLD Center's academic program and supported by other college and campus services and programs[31]. The BOLD Center houses the diversity program in the college.

Engineering GoldShirt Pre-Calculus Background

The ALEKS scores of the Engineering GoldShirt students cover the entire range; the distribution of scores for the fall 2012 cohort is shown in Figure 2.

Figure 2. ALEKS score distribution by demographic
for fall 2012 entering Engineering GoldShirt Program students

Table 1. Pre-calculus for engineers course: units, topics and timeframe.

Unit	Topics
Algebra Fundamentals (12 hours)	Real numbers, exponents and radicals, algebraic expressions
	Rational expressions, equations, modeling with equations
	Inequalities, coordinate geometry, optional: graphing calculators; solving equations and inequalities graphically, lines, making models using variation
Functions (16 hours)	Defining a function, graphs of functions Getting information from the graph of a function
	Average rate of change of functions, transformations of functions, combining functions
	One-to-one functions and their inverses, quadratic functions and models, polynomial functions and their graphs
	Dividing polynomials, real zeros of polynomials, complex numbers, complex zeros and the fundamental theorem of algebra
Exponential Functions and Logarithms (12 hours)	Exponential functions, the natural exponential function, logarithmic functions
	Laws of logarithms, exponential and logarithmic equations
	Modeling with exponential and logarithmic functions
Trigonometry (20 hours)	The unit circle, trigonometric functions of real numbers, trigonometric graphs
	More trigonometric graphs, inverse trigonometric functions and their graphs, modeling harmonic motion
	Angle measure, trigonometry of right triangles, trigonometric functions of angles
	Inverse trigonometric functions and right triangles, the law of sines, the law of cosines, trigonometric identities
	Addition and s.96 -0 0 9.96 226.92 395.4 nd s.96 -0 2 405.48 0.48 24.36 ref535.68 40

Using ALEKS in Pre-Calculus

Incorporation of the ALEKS system enables instructors and students to assess math skills in a variety of categories and for students to learn specific skills in areas that they are not proficient. Students are expected to take ownership of their learning and progress through the modules to improve their understanding of concepts, while also meeting other course expectations. The following ALEKS modules were included in the pre-calculus for engineers course:

- Algebra and Geometry Review (87 topics)
- Functions and Graphs (55 topics)
-

The lower than B- grades earned by the remaining six students (46%) required them to repeat the pre-calculus for engineers course the following semester. Only two of these students re-enrolled in the course in the fall 2012 semester as required, and both earned a B in the fall 2012 semester. Of the remaining four students, three students enrolled in the two-semester calculus I course and one enrolled in the one-semester calculus course, going against the GoldShirt repeat requirement. None of the four earned a B- or better in calculus (three Cs in the yearlong course and one F in the one-semester course), with the consequences that they must all repeat calculus I in the spring 2013 semester.

Figure 3. Final grade distributions from the spring 2012 pilot 1 pre-calculus course and the subsequent fall 2012 calculus grade distributions.

As this GoldShirt cohort moved on, of the 11 students who took calculus 1 in the fall 2012, only twoearned a B - the - osallr(ep)-4(eat)d-6(u)-10(a)-6(x)-4()-2(c) 1 [(2)] -20(e)
Akl(,Tc 0.003 Tw 0 Tw 7.47 0 Td EM(r)3 [(A)ndo.]T

Class Performance Results: Pilot 2

In pilot 2, 29 students enrolled in the pre-calculus for engineers course, of which 22 were Engineering GoldShirt students, including two repeaters from the spring 2012 course. Fourteen students (63.6%) passed with a B- or better. Of the eight students who did not pass with a B- or better, three (13.6%) earned a C or C-, three (13.6%) earned a D or D+ and two earned Fs (9%).

Figure 4. Final grade distribution for the fall 2012 pre-calculus pilot 2.

Of the 14 students who passed with a B- or better, six earned an A or A-, three of them are enrolled in one-semester calculus I, two are enrolled in yearlong calculus I and one left the program. Of the remaining eight students, six are enrolled in yearlong calculus I and two are enrolled in one semester calculus I. Their performances in calculus I will be evaluated at midterm and at spring 2013 semester end. Evaluation results will give a clearer indication of the preparedness of students who took the pilot 2 pre-calculus for engineers course.

The first round of spring 2013 exams in one-semester calculus I and yearlong calculus I for the 13 students is an early indicator of how students are performing in the course. For the one semester calculus I course, the college average was 71% on the first exam. The five students enrolled in one-semester calculus I scored, on average, 74.2% on the first exam, 5% higher than their peers in the college. For the yearlong calculus I course, the college average was 70% on the first exam. The eight students enrolled in yearlong calculus I scored, on average, 73.5% on the first exam, 5% higher than their peers in the college. While this data is preliminary and gains have not been tested for significance, early results are promising.

Of the eight students who did not pass the course with a B- or better, seven enrolled again in pre-calculus for engineers in spring 2013. Although these students mastered some of the concepts in the course, they still have knowledge gaps with major concepts necessary for calculus preparedness. Because of the small number of students, the same instructor who taught pilot 2 has implemented more active learning

Case Studies and Summary of Progress in ALEKS

The

Figure 6. Final ALEKS assessment pie chart for student 1.

Table 2. Student 1's scores in the course grading criteria.

Course Grading Criteria	Score
ALEKS homework	100%
Paper homework	98.2%
WebAssign homework	100%
Midterm exam average	94.7%
Final exam average	96%
Final Grade	**96.7% (A)**

Similar to student 1, student 2 earned a pre-semester, initial ALEKS math placement score of 52/100. Figures 7 and 8 illustrate the initial and final assessment pie charts of student 2 during the pre-calculus for engineers course. The ALEKS system calculated an average of 3.9 hours per week that student 2 dedicated to working on concepts in the system. The student began with 27% mastery on the initial assessment and ended the semester with 100% mastery on the final assessment. Student 2 earned above 90% in ALEKS and WebsdenA5313 039241(s)-1(4(nt)-2(o w)2,8(S)-4(edi)

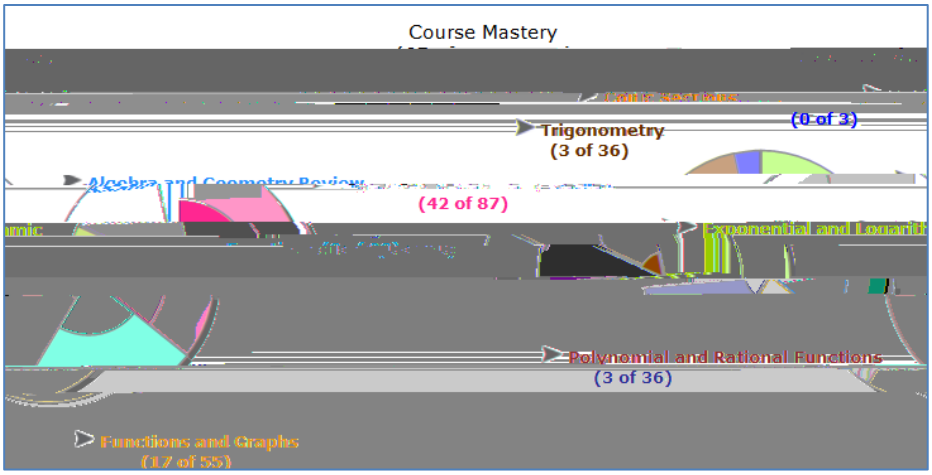

Figure 7. Initial ALEKS assessment pie chart for student 2.

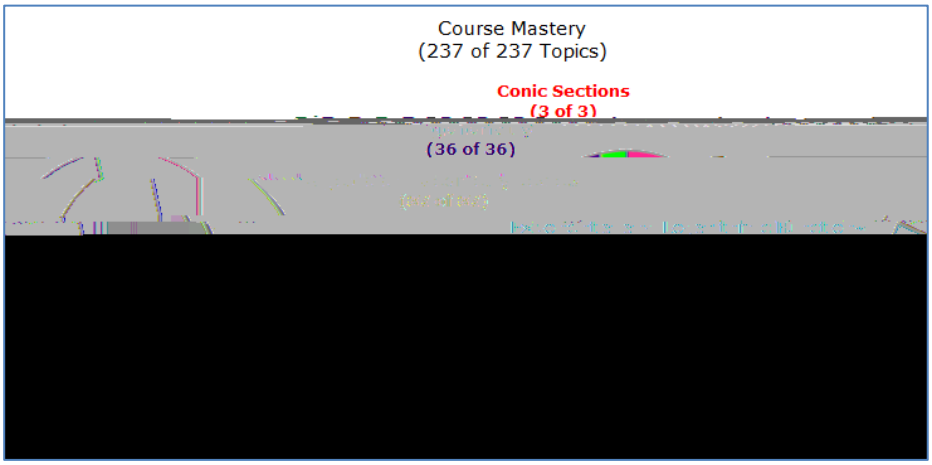

Figure 8. Final ALEKS assessment pie chart for student 2.

Table 3. Student 2's scores in the course grading criteria.

Course Grading Criteria	Score
ALEKS homework	100%
Paper homework	82.8%
WebAssign homework	99.5%
Midterm exam average	75.37%
Final exam average	84%
Final Grade	**83.87% (B)**

Student 3 earned a pre-semester, initial ALEKS math placement score of 18/100. The ALEKS system calculated an average of 4.1 hours of week that the student dedicated to working on concepts in the system. Figures 9 and 10 show student 3's ALEKS pie charts for the initial and final assessments that were ascertained in the pre-calculus course for engineers. The student began with 19% mastery on the initial assessment and ended the semester with 73% mastery on the final assessment. Table 4 shows the inconsistency of student 3's grades in ALEKS, paper and

WebAssign homework, including midterm and final exam scores that were below the class average. Instead of demonstrating strong work in the multiple course components, the inconsistent performance of student 3 resulted in a D+ course grade.

Figure 9. Initial ALEKS assessment pie chart for student 3.

Figure 10. Final ALEKS assessment pie chart for student 3.

Table 4. Student 3's scores in the course grading criteria.

Cours

Looking across the class performance as a whole, Figures 11 and 12 show ALEKS performance for the course.

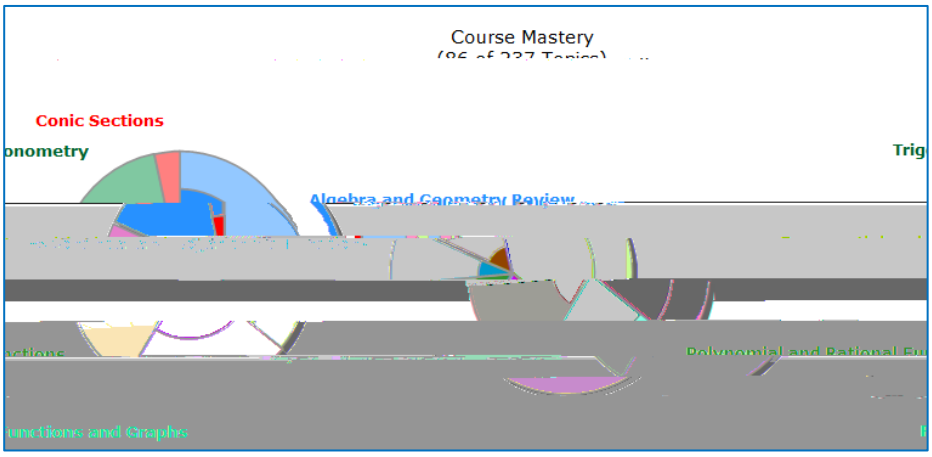

Figure 11. Initial ALEKS assessment pie chart for overall class performance.

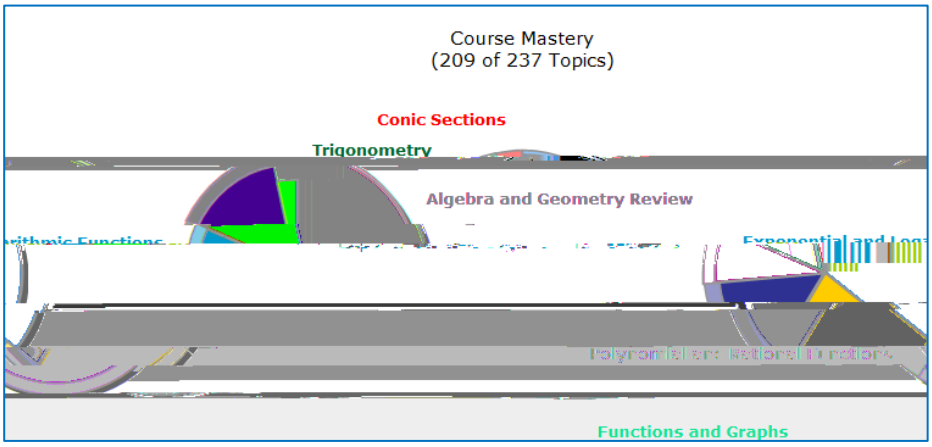

Figure 12. Final ALEKS assessment pie chart for overall class performance

Table 5 reflects the initial and post assessment results and percent increase for each topic. These results reflect significant growth for the class as a whole for all topics.

Table 5. Class performance—mastery of ALEKS topics: initial and final assessment.

ALEKS Objectives/Topics	Class Initial Assessment	Class Final Assessment	% Increase
Algebra and geometry review	58%	95%	**64%**
Functions and graphs	40%	86%	**115%**
Polynomial and rational functions	16%	82%	**413%**
Exponential and logarithmic functions	14%	83%	**493%**
Trigonometry	12%	86%	**617%**
Conic sections	18%	71%	**294%**

Assessment Results

Data were collected from 29 participants in the pilot 2 pre-calculus for engineers course. The course included 28% women, 52% URM students, and 83% of students were in their first year of engineering. Course assessments included a post-survey, focus group and case study. The post-survey is composed of quantitative questions asking about the impact of the course on mastery of pre-calculus as well as experiences with course components. The focus group was conducted at the course end and had the class divide into teams that generated strengths and suggestions for improvement to the course.

The results of student self-ratings of their pre-calculus skills are listed in Table 6. While only 21.5% of students felt themselves to be moderately or highly skilled at the start of the course, 85.7% of students found themselves to be moderately or highly skilled at the end of the course, a gain of 298%. While the number of data points was too small to test for significance in this pilot course, gains in self-rated skills were substantial. The post self-ratings were significantly and positively correlated with course grades ($r = .69$, $p < .05$) indicating that students were aware of their mastery levels in the class. The course grade was also positively correlated with ratings of the value of instructor support ($r = .59$, $p < .05$) highlighting the value of the instructor in course

Figure 13. Impact of course components on pre-calculus learning.

Focus group results showed the course to be well-paced and comprehensive. Students like the small class size. The workload, not including ALEKS, was described as "about right" at 6-10 hours per week. However, students spent up to 20 hours per week on ALEKS assignments. The teacher was rated as enthusiastic and lectures were easy to follow. Suggestions for improvement were mainly around better incorporation of the online ALEKS software into the class. Students wanted milestone dates for module completion so that ALEKS work did not pile up on them. Students also requested that ALEKS topics better line up with weekly course topics. Use of this software was new, and students recognized that this was the first semester of its implementation and expressed hope for greater integration in the future.

Students completed a survey requesting feedback on the course. Several themes emerged from the following questions, along with illustrative student quotes:

<u>Question 1:</u> What did you like most about the pre-calculus for engineers course?
Emerging Themes: The instructor, the pace of the course and learning/preparing for calculus

> *"One thing that I have liked about the pre-calculus*

"The way that everything we learned leads up to the next thing and is focused on the topics' applications to calculus. Also, the teacher has a good teaching style and works to have students understand the topics."

Question 2: What was most difficult about the pre-calculus course?
Emerging Themes: ALEKS, workload and pace

"… the work load, it was a lot of homework to keep up with."

"The ALEKS online assignments, because the large amounts of topics due at one time and also because when one makes a mistake in ALEKS, they have to do an extra similar problem, and sometimes these can compound."

"I would say the class could improve by having weekly check-ups. Sometimes people learn at different paces and need to be checked on to see if they are correctly implementing the concept."

Question 3: Do you have any suggestions for improvement to the pre-calculus course?
Emerging Themes: Better integration and consistency of homework (WebAssign, ALEKS and written), remove ALEKS, offer more study guides, sessions and check-ups

"I would choose between either ALEKS or WebAssign; having both not only is extremely stressful, but also confusing since both sources have different grading methods and teach differently."

"My suggestions are to have review packets

explain concepts and their problem solving strategies to the instructor. These students will also continue working in ALEKS to achieve mastery of all course modules.

Spring 2013 calculus I performance will be tracked for students who successfully completed the pilot 2 pre-calculus for engineers course. Their performance will indicate the effectiveness of the pilot 2 pre-calculus course, and help to guide changes necessary to improve the course for fall 2013 (pilot 3).

The ALEKS math placement test results indicate that a critical mass of freshman students enter the college who are not yet prepared for calculus I (see Figure 1). An initiative is underway to explore hosting multiple sections of the pre-calculus for engineers course to enroll these students who need calculus I preparation.

Bibliography

1. University of Texas Austin Cockerell School of Engineering, “Calculus Readiness Requirement,” http://www.engr.utexas.edu/undergraduate/admission/calculus, 2012.

2. Wheeler, E., “Calculus Expectations and Retention,” 2010 Proceedings of the ASEE North Central Sectional Conference, Pittsburgh, PA.

3. Jovanovic, N. S., “The Meaning of ACT Mathematics Scores: Implications for Engineering Programs, K-12 Schools, a(ke)-u2(ool)7d S65 0 TCn98. S.useel,oceedins (“)12(o)-4(f)i0(o)-0(o)- 4(f)n3(cat)3(iS)4(“ C.012 0s)5(cee)8(s)euiceecemC